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RESEARCH ARTICLE - SYSTEMS ENGINEERING

Remedial Measures to Lessen the Effect of Imprecise Measurement with Linearly Increasing Variance on the Performance of the MAX-EWMAMS Scheme

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Abstract Simultaneous monitoring of process mean and variability by a single control chart has been increasingly taken into consideration in recent years. However, the effect of imprecise measurements on the performances of some existing control schemes has been neglected. In this paper, the effect of measurement errors with linearly increasing variance on the detecting and diagnosing capability of the MAX-EWMAMS control chart is first investigated in Phase II monitoring. The results obtained using simulation studies show that the measurement errors affect the two performances of the chart, significantly. Then, two remedial measures including the ranked set sampling approach and using a larger sample size are proposed. The simulation results confirm that both the remedial measures compensate for the effect of measurement errors, adequately. A real-data example is also given to illustrate the effect of measurement errors on the rate of false alarm.

Keywords MAX-EWMAMS control chart · Measurement error · Ranked set sampling · Source of signal · Phase II

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1 Introduction

In the past decades, the application of control charts has been widespread in manufacturing and service sectors. Using control charts leads to improving the quality of products and/or services by distinguishing between common and assignable causes of variations. Most studies in statistical process monitoring are provided under the assumption that the measurements are precise. However, exact measurement is a rare phenomenon in any environment where the human involvement is evident [1]. In other words, errors due to instruments and operators commonly exist in practice even with highly sophisticated advanced measuring instruments. The measurement errors may affect the performance of control charts in two ways: (1) it can adversely affect the performance of control charts in detecting out-of-control states, and (2) it can increase the rate of false alarms. Khurshid and Chakraborty [2] stated that the sources of variations might be due to the inherent variability in the process and the errors due to the measurement instruments.

Recently the effect of measurement errors on the performances of various schemes proposed to monitor different processes has been addressed by several researchers such as [3–14]. For detailed information concerning the effect of measurement errors on the performance of control charts refers to the review paper presented by Maleki et al. [15]. Most of the above-mentioned researchers focused on evaluating the effect of measurement errors on the performance of a scheme to monitor either the process mean or its variability. Nonetheless, simultaneous monitoring of process mean and variability has received a great deal of attention in recent years due to two main reasons; (1) both the mean and variability may shift at the same time, and (2) a change in the process variability can affect the control limits of the charts for monitoring the process mean. For more information concerning

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the simultaneous monitoring schemes, interested readers are referred to [16–22]. Moreover, McCracken and Chakraborti [16] provided a review of simultaneous monitoring procedures available in the literature until 2011.

To the best of the authors' knowledge, research conducted in the context of the simultaneous monitoring of process mean and variability has neglected the effect of imprecise data obtained under measuring systems, due to either human fault or equipment error. In addition, most studies in measurement errors have assumed a constant variance for the measurement error term. In some practical problems, however, the variance of the measurement error term depends on the mean level of the process. It has been proven in the literature that the capability of the control charts in detecting out-of-control situations is directly affected by the magnitude of the process variability [2]. However, only a few remedial measures have been recommended in the literature to compensate for the effect of measurement errors on detecting capability of control charts. Therefore, proposing some novel remedial approaches to cover such effects is inevitable.

In this paper, the effect of measurement errors on the performance of one of the most common approaches in the literature for simultaneous monitoring of the process mean and variability, called the maximum exponentially weighted moving average and mean-squared deviation (MAX-EWMAMS) control chart, is investigated. This effect is studied for measurement errors with linearly increasing variance. In other words, the assumption of constant variance for the measurement error term is relaxed, where it is assumed that this variance linearly relates to the process mean level. Through a numerical example, we will first show that the presence of measurement errors can affect the detecting and diagnosing capability of the MAX-EWMAMS control chart adversely. Then, we extend and utilize two remedial measures to compensate for the effect of measurement errors. Through an illustrative numerical example, we will show that the remedial approaches including a ranked set sampling (RSS) approach and using a larger sample size reduce the effect of measurement errors. We assume that the model parameters are known based on historical data, i.e., the investigation and the proposals are designed for a Phase II analysis.

To structure the literature review of the process monitoring and to demonstrate differences between this study and other works, a systematic state-of-the-art survey is depicted in Table 1 in terms of the type of error variance for the measurement error term, the parameters (mean, variance or both) that are monitored, the remedial approach, and the diagnosis procedure. It is concluded from Table 1 that (1) most of the studies have considered constant variances for measurement errors and ignore joint monitoring of process mean and variance, (2) only a few papers are associated with diagnosing the source of changes, (3) most of the papers have not introduced any remedial approach to compensate for the effect of measurement errors on detecting capability of control charts.

The rest of this paper is organized as follows: In Sect. 2, the problem is briefly defined and the notations are introduced. The MAX-EWMAMS control chart is discussed in Sect. 3. Section 4 contains an extension of the MAX-EWMAMS control chart using a model with an additive covariate error term and discusses the diagnosing procedure. The effect of measurement errors with linearly increasing variance on the performance of the MAX-EWMAMS control chart is assessed by simulation studies in Sect. 5. A sensitivity analysis is also performed in Sect. 5 to evaluate the detecting and diagnosing capability of the MAX-EWMAMS control chart in the presence of measurement errors. In Sect. 6, the ranked set sampling (RSS) approach is extended to compensate for the effect of measurement errors on the performance of the MAX-EWMAMS control chart. In Sect. 7, a real-life example is given to illustrate the effect of gauge measurement errors on the rate of false alarm. Finally, the findings are concluded in Sect. 8, where recommendations for future study are provided.

2 Problem Definition

Consider a process of interest with the in-control mean and variance denoted by μ_0 and σ_0^2, respectively. It is assumed that the observations are independent and follow $N\left(\mu_0, \sigma_0^2\right)$. Observations are gathered in a rational subgroup of size n where the jth observation at the tth sampling point is denoted by X_{tj}. In other words, $\mathbf{X_t} = [X_{t1}, X_{t2}, ..., X_{tn}]$ denotes the vector of tth random sample which contains the true observations of the quality characteristic under investigation. We are to design a MAX-EWMAMS scheme in the presence of measurement errors in order to monitor the mean and the variance of the process, simultaneously. The notations used to formulate the problem are present in Table 2.

3 The MAX-EWMAMS Control Chart

In this section, the MAX-EWMAMS scheme for simultaneous monitoring of the process mean and variability, which plots only one statistic at a time, is briefly explained. This scheme utilizes an exponential weighted moving average (EWMA) and an exponential weighted mean square (EWMS) statistics to monitor the mean and the variance of a process, simultaneously. The EWMA statistic for monitoring the process mean at tth sample with the smoothing parameter λ selected in the range [0,1] is given by

$$Z_t = \lambda \bar{X}_t + (1 - \lambda) Z_{t-1}, \tag{1}$$

Table 1 Comparing existing studies with the proposed methodology

Research	Error variance		Remedial approach	Monitoring		Diagnosis	Control chart
	Constant	Linearly increasing		Mean or variability	Mean and variability		
Yang and Yang [3]	✓			✓			Shewhart chart/cause selecting chart
Yang et al. [4]	✓			✓			EWMA chart/cause selecting chart
Maravelakis [5]	✓			✓			CUSUM chart
Memar and Niaki [23]					✓	✓	MAX-EWMAMS chart
Moameni et al. [7]	✓			✓			$\tilde{X} - \tilde{R}$ fuzzy chart
Maravelakis [8]	✓	✓	✓	✓			CUSUM chart
McCracken and Chakraborti [16]					✓	✓	Max chart/Distance chart
Hu et al. [9]	✓		✓	✓			Synthetic $\bar{X}$ chart
Khurshid and Chakraborty [2]	✓			✓			chart for standardized zero truncated binomial variables
Riaz [1]	✓		✓	✓			Shewhart charts
Chowdhury et al. [17]					✓	✓	Shewhart-Cucconi (SC) chart
Park [18]					✓		Semi-circle chart/Max chart/GLR chart/Fisher chart/UI chart/Liptak chart
Haq et al. [10]	✓	✓	✓	✓			EWMA chart
Hu et al. [11]	✓	✓	✓	✓			adaptive $\bar{X}$ chart
Noorossana and Zerehsaz [12]	✓			✓			EWMA-3 chart/ EWMA/R chart/T^2 chart
Chowdhury et al. [19]					✓		cumulative sum-Lepage (CL) chart
Afolabi et al. [20]					✓		Lepage-type change-point (LCP) chart
Prajapati and Singh [21]					✓		modified $\bar{X}/R$ chart
Maleki and Amiri [22]				✓	✓		artificial neural network
Abbasi [13]	✓		✓	✓			EWMA chart
Yeong et al. [14]	✓		✓	✓			coefficient of variation chart
This paper		✓	✓	✓	✓	✓	MAX-EWMAMS chart

Table 2 Notation and definitions

Notation	Description
t	Index of a sample
j	Index of an observation
μ_X	Mean of the quality characteristic under investigation
μ_0	In-control mean of the quality characteristic under investigation
σ_X^2	Variance of the quality characteristic under investigation
λ	Smoothing parameter of the control chart
df	Degree of freedom of a chi-square distribution
X_{tj}	The actual value of jth observation in tth sample
$\bar{X}_t$	The sample mean in tth sample
S_t^2	The EWMS statistic for tth sample
Z_t	The EWMA statistic based on tth sample
U_t	The standardized EWMA statistic for tth sample
V_t	The standardized EWMS statistic based on tth sample
M_t	The joint monitoring statistic for tth sample
A	The intercept parameter of the model that includes the covariate term
B	The slope parameter of the model involving the covariate term
C	The intercept parameter of the line modeling the error variance
D	The slope parameter of the line modeling the error variance
Y_{tj}	The measured value of jth observation in tth sample
$\bar{Y}_t$	The sample mean of Y's in tth sample
$S_t'^2$	The EWMS statistic corresponding to Y's
σ_y^2	The variance of the measured quality characteristic under covariate model
Z_t'	The EWMA statistic corresponding to Y's
U_t'	The standardized EWMA statistic corresponding to Y's
V'	The standardized EWMS statistic corresponding to Y's
M_t'	The joint monitoring statistic corresponding to Y's

where $\bar{X}_t$ is the sample mean at time t and $Z_0 = \mu_0$. The EWMS statistic for monitoring the process variability at tth sample with the smoothing parameter λ is obtained based on Eq. 2.

$$S_t^2 = (1-\lambda)S_{t-1}^2 + \lambda \sum_{j=1}^{n} \frac{(X_{tj}-\mu_0)^2}{n}, \tag{2}$$

where $S_0^2 = \sigma_0^2$. It can be statistically checked that the expected value and the variance of S_t^2 are obtained according to Eqs. 3 and 4, respectively:

$$E[S_t^2] = \sigma_0^2, \tag{3}$$

$$\mathrm{Var}[S_t^2] = \frac{2\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma_0^4. \tag{4}$$

When the observations are independent and Normally distributed and $t \to \infty$, we have:

$$S_t^2/\sigma_0^2 \to \frac{\chi_{df}^2}{df};\ df = n(2-\lambda)/\lambda. \tag{5}$$

Obviously $Z_t \sim N(\mu_0, \frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\sigma_0^2\right]$. Then, the U_t-statistic with the standard Normal distribution used to monitor the process mean is turned to be:

$$U_t = \frac{(Z_t-\mu_0)}{\sqrt{\frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\sigma_0^2\right]}}. \tag{6}$$

For monitoring the process variability, the V_t-statistic is used as follows:

$$V_t = \phi^{-1}\left[\Pr\left(\chi_{df}^2 \le \frac{df \times S_t^2}{\sigma_0^2}\right)\right]. \tag{7}$$

Note that according to Ostadsharif Memar and Niaki [23], V_t approximately follows the standard Normal distribution. Finally, the MAX-EWMAMS statistic at the sample point t; $t = 1, 2, \ldots$ is defined as follows:

$$M_t = \max\{|U_t|, |V_t|\}. \tag{8}$$

Note that the MAX-EWMAMS control chart only has an upper control limit (UCL) because $M_t \geq 0$.

4 Modification of the MAX-EWMAMS Scheme in the Presence of Measurement Error

The additive covariate model that expresses the measured value of jth observation in tth sample is shown in Eq. (9) [24]:

$$Y_{tj} = A + BX_{tj} + \varepsilon_{tj}, \tag{9}$$

where A and B are intercept and slope constants of a model involving a covariate term and ε_{tj} is the random error term which is independent of X_{tj}. Most researchers assumed that ε's follow Normal distributions with mean 0 and constant variances. However, in some applications, the error variance is not constant and may depend on the process level. In this paper, it is assumed that the variance of the error term changes linearly with the process mean, i.e., $\varepsilon \sim N(0, C + D\mu_X)$. Consequently, Y's are Normally distributed with the following parameters:

$$Y_{tj} \sim N(A + B\mu_X, B^2\sigma_X^2 + C + D\mu_X). \tag{10}$$

The EWMA statistic for monitoring the process mean in the presence of the measurement errors with linearly increasing variance is:

$$Z'_t = \lambda \bar{Y}_t + (1 - \lambda)Z'_{t-1}, \tag{11}$$

where $Z'0 = A + B\mu_0$ and $\bar{Y}_t = \sum_{j=1}^{n} Y_{tj}/n$. It can be shown that Z'_t follows a Normal distribution as:

$$Z'_t \sim N\left(A + B\mu_0, \left(\frac{\lambda}{n(2-\lambda)}\right)[1 - (1 - \lambda)^{2t}] \times (B^2\sigma_0^2 + C + D\mu_0)\right). \tag{12}$$

Then, in the presence of the measurement errors, the standardized statistic for monitoring the process mean is:

$$U'_t = \frac{Z'_t - (A + B\mu_0)}{\sqrt{\frac{\lambda}{(2-\lambda)}\left[1 - (1 - \lambda)^{2t}\right] \times \frac{B^2\sigma_0^2 + C + D\mu_0}{n}}}. \tag{13}$$

For monitoring process variability when the measurements are imprecise, Eq. (2) is rewritten as:

$$S_t'^2 = (1 - \lambda)S_{t-1}'^2 + \lambda \sum_{j=1}^{n} \frac{\left(Y_{tj} - (A + B\mu_0)\right)^2}{n}. \tag{14}$$

Then, the chart statistic for monitoring process variability with approximately standard Normal distribution is:

$$V'_t = \phi^{-1}\left[\Pr\left(\chi^2_{df} \leq \frac{df \times S_t'^2}{\sigma_Y^2}\right)\right]. \tag{15}$$

Since both U'_t and V'_t follow the standard Normal distribution, the chart statistic for simultaneous monitoring of the process mean and variability in the presence of imprecise measurements will be obtained as:

$$M'_t = \max\{|U'_t|, |V'_t|\}. \tag{16}$$

Since Eq. (16) guarantees that $M'_t \geq 0$, an upper control limit is only required to monitor the process. The upper control limit is set such that the in-control average run length (ARL_0) becomes a predetermined value.

The most important issue after receiving an out-of-control signal, i.e., $M'_t >$ UCL, is to diagnose the source of the signal. In this regard, the following rules are utilized:

1. If $|U'_t| >$ UCL and $|V'_t| \leq$ UCL, the process mean is responsible for the signal.
2. If $|V'_t| >$ UCL and $|U'_t| \leq$ UCL, the process variability is responsible for the signal.
3. If $|U'_t| >$ UCL and $|V'_t| >$ UCL, both the mean and the variability are responsible for the signal.

Assume that a given simultaneous shift ($\mu_X = \mu_0 + \delta\sigma_0, \sigma_X = \psi\sigma_0$) leads to an out-of-control signal at tth sample. In such situation, the probability that both $|U'_t|$ or $|V'_t|$ statistics jointly exceed the UCL ($P(U'_t >$ UCL$, V'_t >$ UCL$|\mu_X, \sigma_X)$) is less than the probability that only one of them exceeds the UCL, $\big(P(|U'_t| >$ UCL$, |V'_t| \leq$ UCL$|\mu_X, \sigma_X) + P(|V'_t| >$ UCL$, |U'_t| \leq$ UCL$|\mu_X, \sigma_X)\big)$. Therefore, the performance of the MAX-EWMAMS control chart in diagnosing the source of the signal under simultaneous shifts will be less compared to the ones obtained in diagnosing the mean or the variance shifts, separately.

5 Effect of the Measurement Errors

In this section, the effect of the measurement error with a linearly increasing variance on the performance of the MAX-EWMAMS control chart is evaluated through 10,000 simulation replicates. We assume that $X \sim N(5, 1)$ with $A = 0$ and $B = 1$ in the additive covariate error model. In all simulation experiments, the UCL value of the MAX-EWMAMS control chart is set such that $ARL_0 \approx 200$. The step shifts of the process mean and variance are denoted by $(\mu_0 + \delta\sigma_0)$ and $(\psi\sigma_0)$, respectively. Table 3 contains ARL

Table 3 ARLs when $n = 10$, $\lambda = 0.2$

UCL (δ, ψ)	2.8710 (C,D)	2.8833	2.8844	2.8837	2.889
	(0,0)	(0,1)	(0,2)	(1,1)	(3,1)
(0,1)	199.992	199.321	199.303	200.817	199.867
(0.25,1)	13.320	61.600	85.477	68.563	84.584
(0.5,1)	3.987	18.628	29.841	21.600	26.442
(0.75,1)	2.078	8.795	14.520	10.065	13.102
(1,1)	1.440	5.377	8.592	6.198	7.851
(1.25,1)	1.143	3.755	5.882	4.247	5.269
(1.5,1)	1.031	2.930	4.388	3.185	3.925
(0,0.25)	3.000	83.985	196.448	106.009	151.386
(0,0.5)	3.614	125.013	198.780	135.181	192.382
(0,1.1)	33.433	146.598	184.205	161.702	187.585
(0,1.2)	10.927	109.810	160.490	115.676	137.622
(0,1.3)	5.884	75.076	125.234	90.020	111.515
(0.25,0.25)	10.804	66.645	108.254	79.417	98.372
(0.25,0.5)	14.043	74.822	110.32	85.582	101.415
(0.25,1.1)	10.597	50.229	70.889	61.392	73.423
(0.25,1.2)	7.306	41.153	61.781	51.304	62.439
(0.5,0.25)	3.991	20.907	35.831	24.218	31.826
(0.5,0.5)	4.005	21.293	36.716	24.496	32.717
(0.5,1.1)	3.966	17.339	26.909	21.253	27.771
(0.5,1.2)	3.545	16.884	25.694	19.610	24.994
(0.75,0.25)	2.063	9.285	15.750	10.820	13.385
(0.75,0.5)	2.091	9.626	16.140	11.092	13.770
(0.75,1.1)	2.138	8.303	13.721	10.141	12.883
(0.75,1.2)	2.033	8.265	12.976	9.627	12.834
(1,0.25)	1.390	5.687	9.132	6.465	8.015
(1,0.5)	1.403	5.753	9.251	5.522	7.722
(1,1.1)	1.471	5.327	8.449	6.072	7.581
(1,1.2)	1.470	5.303	8.238	5.925	7.556
(1.25,0.25)	1.091	3.852	6.390	4.422	5.280
(1.25,0.5)	1.115	3.880	6.427	4.554	5.511
(1.25,1.1)	1.174	3.610	5.729	4.310	5.317
(1.25,1.2)	1.172	3.650	5.521	4.263	5.209

values when $n = 10$ and $\lambda = 0.2$ under different values of (C, D). The ARLs are presented at different values of δ and ψ. Note that the process mean is in-control if $\delta = 0$ while the process variability is in-control if $\psi = 1$. One can see from Table 3 that measurement errors with a linearly increasing variance adversely affect the performance of the MAX-EWMAMS control chart in detecting all process changes. In addition, as both parameters C and D increase, the ARLs under mean shifts, variance shifts and simultaneous shifts in both tend to increase.

The performance of the MAX-EWMAMS control chart to diagnose the source of the signal under different values of (C, D) is evaluated in Table 4. In Table 4, $n = 10$, $\lambda = 0.2$

Table 4 Correct diagnosis percentage when $n = 10$, $\lambda = 0.2$

(δ, ψ)	(C,D)				
	(0,0)	(0,1)	(0,2)	(1,1)	(3,1)
(0.25,1)	95.8	74.7	66.0	74.1	73.1
(0.5,1)	97.5	85	78.3	88.4	86.6
(0.75,1)	96.2	86.9	81.8	87.4	91.1
(1,1)	91.9	85.2	81.3	87.9	91.2
(1.25,1)	80.1	85.9	81.3	86.3	91.3
(0,0.25)	100	90.1	68.2	88	75.9
(0,0.5)	100	84.1	60.1	75.7	68.2
(0,1.1)	82.7	55.4	52.4	53.3	52.7
(0,1.2)	90.2	62.3	57.1	64.1	57.6
(0,1.3)	91.0	72.1	60.5	69.7	63.6
(0.5,0.25)	51	1	0.8	0.7	0.9
(0.5,0.5)	51.7	1.6	1.5	0.6	1.1
(0.5,1.25)	24.6	7.2	4.1	5.0	4.2
(0.5,1.5)	32.7	11.1	5.5	6.3	6.0
(0.5,2)	33.1	11.5	7.3	10.9	7.9
(0.75,1.25)	29.7	9.3	6.3	7.9	6.1
(0.75,1.5)	46.9	12.4	10.5	9.7	7.9
(0.75,2)	47.4	16.2	12.1	15.6	12.0

and the results are obtained in terms of the correct diagnosis percentage (CDP) criterion. The results in Table 4 show that increasing both parameters C and D results in increased measurement errors affecting CDPs. It is seen that the diagnosing performance of the MAX-EWMAMS scheme in detecting mean shifts and variance shifts is satisfactory. However, for simultaneous shifts, the MAX-EWMAMS control chart under covariate model does not accurately diagnose the source of the signal.

In the rest of this section, a sensitivity analysis with respect to the sample size parameter when $C = 1$, $D = 1$, $\lambda = 0.2$ is performed. The *ARL* values under mean and variance shifts for different values of the parameter n are displayed in Table 5 while for joint shifts the ARL values are depicted in Fig. 1. Both Table 5 and Fig. 1 indicate that as the sample size increases, the effect of measurement error on the capability of the MAX-EWMAMS control chart in detecting mean shifts, variance shifts, and joint shifts decreases. In addition, Table 6 confirms that when the parameter n increases, the diagnosing capability of the MAX-EWMAMS scheme under measurement error with linearly increasing variance is improved. This suggests using a larger sample size to compensate for the effect of measurement errors on detecting and diagnosing capability of the MAX-EWMAMS control chart effectively. It is worth mentioning that, although increasing the sample size improves the statistical features of the MAX-EWMAMS control chart under measurement errors, it is associated with a higher sampling cost.

6 A Rank Set Sampling Approach

One of most efficient methods in decreasing the effect of measurement errors on the performance of control charts is the *rank set sampling* (RSS) method. The effectiveness of this method has been shown in some studies such as Al-Nasser and Al-Rawwash [25]. In addition, Muttlak and Al-Sabah [26] developed two modified versions of the RSS method, namely median ranked set sampling (MRSS) and extreme ranked set sampling (ERSS). They indicated better performance of MRSS- and ERSS-based charts compared to the usual control charts based on simple random sampling (SRS) approach in terms of the *ARL* criterion.

Table 5 ARL comparison between different sample sizes under mean and variance shifts when $C = 1$, $D = 1$, $\lambda = 0.2$

UCL	2.8837	2.8850	2.8854	2.8858
(δ, ψ)	n			
	10	12	15	20
(0,1)	200.817	199.158	200.261	199.888
(0.25,1)	68.563	60.898	49.85	41.757
(0.5,1)	21.600	17.81	14.867	11.474
(0.75,1)	10.065	8.486	7.16	5.573
(1,1)	6.198	5.288	4.570	3.534
(1.25,1)	4.247	3.818	3.079	2.467
(1.5,1)	3.185	2.782	2.390	1.988
(0,0.25)	106.009	92.529	72.784	54.615
(0,0.5)	135.181	129.534	110.401	82.274
(0,1.1)	161.702	158.584	154.654	152.712
(0,1.2)	115.676	113.583	112.997	101.855
(0,1.3)	90.020	80.903	67.892	62.893

In this section, a remedial approach based on the RSS approach is proposed to lessen the effect of the measurement errors with linearly increasing variance on the performance of the MAX-EWMAMS control chart. To do this, the following steps are first taken:

1. Select n random samples of size n units from the process.
2. Rank the units within each sample with respect to the measured quality characteristic.

Table 6 Correct diagnosis percentages for different values of n when $C = 1$, $D = 1$, $\lambda = 0.2$

(δ, ψ)	n			
	10	12	15	20
(0.25,1)	74.1	78.2	81	85.3
(0.5,1)	88.4	89.5	90.5	91.5
(0.75,1)	87.4	93	93	94.5
(1,1)	87.9	90.6	91.1	91.5
(1.25,1)	86.3	88.7	90.5	91.9
(0,0.25)	88	88.2	89.8	93
(0,0.5)	75.7	81.2	83.3	87.4
(0,1.1)	53.3	56.1	57.1	57.2
(0,1.2)	64.1	62.6	64.3	67
(0,1.3)	69.7	71.1	71.7	77.7
(0.5,0.25)	0.7	0.7	0.7	0.7
(0.5,0.5)	0.6	0.6	0.6	0.6
(0.5,1.25)	5.0	5.3	5.6	6.5
(0.5,1.5)	6.3	7.4	8	8.7
(0.5,2)	10.9	9.5	10.6	12.1
(0.75,1.25)	7.9	8.2	9	9.6
(0.75,1.5)	9.7	11.7	13.5	14.3
(0.75,2)	15.6	17	17.7	19.8

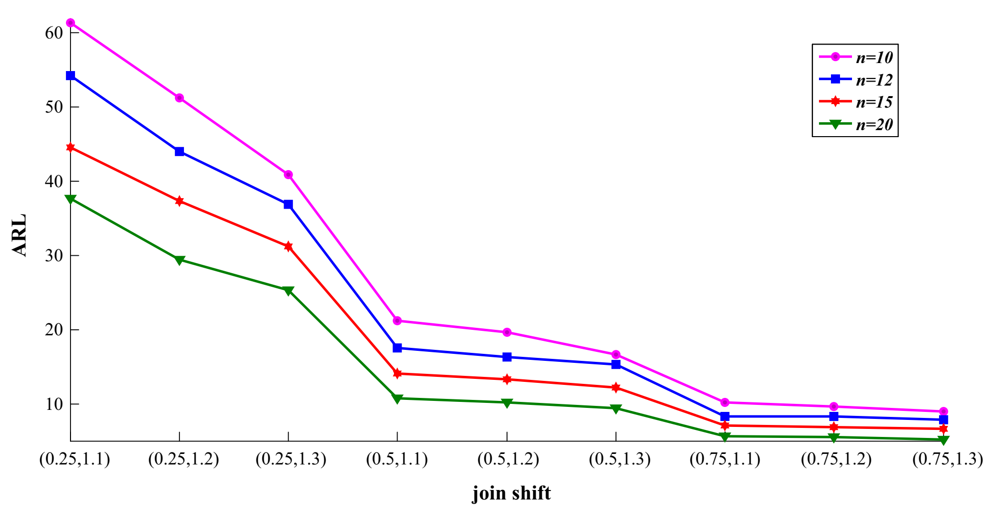

Fig. 1 ARL comparison between different values of n under simultaneous shifts when $C = 1$, $D = 1$, $\lambda = 0.2$

3. The smallest ranked measured quality characteristic is selected from the first set. Similarly, the second smallest ranked quality characteristic is selected from the second set. The procedure continues and the largest ranked quality characteristic is selected from the nth set
4. This completes one cycle of a ranked set sample of size n.

Let $\{Y_{11}, Y_{12}, ..., Y_{1n}\}; \{Y_{21}, Y_{22}, ..., Y_{2n}\}; ...; \{Y_{n1}, Y_{n2}, ..., Y_{nn}\}$ be n independent simple random samples from the measured quality characteristic with probability density function $f(y)$ with finite mean μ_Y and variance σ_Y^2. Let $Y_{i(i:n)}$ denotes the ith ordered statistic from the ith sample of size n. The ranked set sample of size n with respect to the measured units at time t is expressed as follows:

$$\mathbf{Y}_t^{\mathrm{RSS}} = \left\{Y_{t,1(1:n)}, Y_{t,2(2:n)}, ..., Y_{t,n(n:n)}\right\} \tag{17}$$

The tth standardized statistic for monitoring the process mean is rewritten according to Eq. (17):

$$U_t'' = \frac{Z_t'' - (A + B\mu_0)}{\sqrt{\frac{\lambda}{(2-\lambda)}\left[1 - (1-\lambda)^{2t}\right] \times \frac{B^2\sigma_0^2 + C + D\mu_0}{n}}}, \tag{18}$$

where:

$$Z_t'' = \lambda \bar{Y}_t^{\mathrm{RSS}} + (1-\lambda) Z_{t-1}''. \tag{19}$$

In Eq. (19), $\bar{Y}_t^{\mathrm{RSS}} = \sum_{j=1}^{n} Y_{t,j(j:n)}/n$ and $Z_0'' = A + B\mu_0$.

Then, the modified chart statistic for monitoring the process variability is:

$$V_t'' = \phi^{-1}\left[\Pr\left(\chi_{df}^2 \le \frac{df \times S_t''^2}{\sigma_Y^2}\right)\right], \tag{20}$$

where:

$$S_t''^2 = (1-\lambda) S_{t-1}''^2 + \lambda \sum_{j=1}^{n} \frac{\left(Y_{t,j(j:n)} - (A + B\mu_0)\right)^2}{n}. \tag{21}$$

Finally, the modified statistic for simultaneous monitoring of the mean and the variance of the process under linearly increasing variance of the error term is given by:

$$M_t'' = \max\{|U_t''|, |V_t''|\}. \tag{22}$$

The flowchart of the modified MAX-EWMAMS control chart under measurement errors that uses the RSS approach is illustrated in Fig. 2.

Now, to illustrate the effectiveness of the RSS approach in reducing the effect of measurement errors on the performance of the MAX-EWMAMS control chart, the ARL values of the RSS-Max-EWMAMS chart under measurement errors are summarized in Table 7. The results in this table indicate that in almost all shifts, the RSS approach improves the detecting capability of the MAX-EWMAMS control chart. Similar to the first remedial approach, although RSS improves the statistical capability of the MAX-EWMAMS scheme, it results in a higher sampling cost. Consequently, in processes with negligible sampling costs both remedial approaches are recommended for sampling strategies.

In addition, the ARLs of the proposed control chart under simple random sampling (SRS) and ranked set sampling (RSS) schemes under $n = 10, \lambda = 0.2, C = 1$ and $D = 1$ are present in Fig. 3. It can be easily seen from this figure that when measurement errors exist, using the RSS approach leads to a better capability of the MAX-EWMAMS control chart in comparison with the SRS.

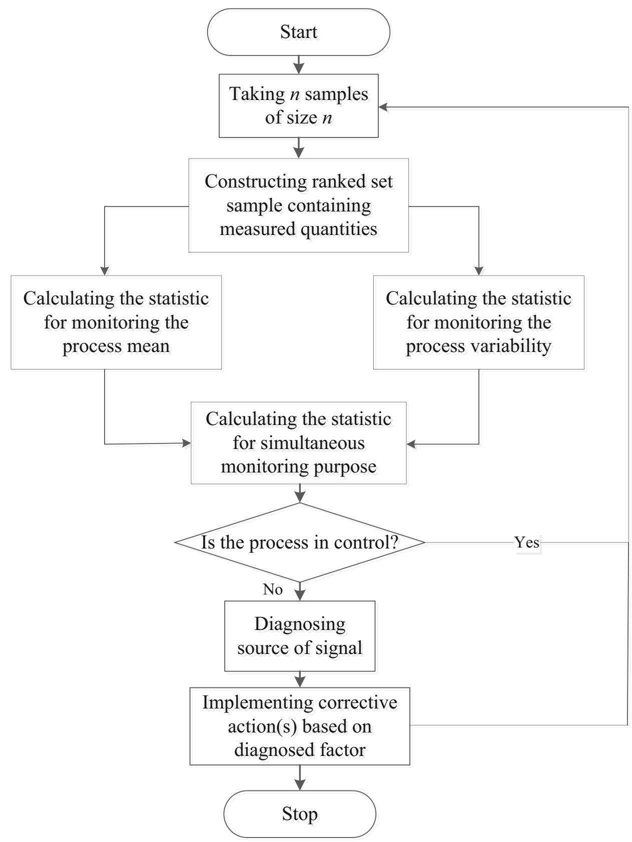

Fig. 2 Flowchart of the proposed method under the RSS approach

7 Real-Life Example

In this section, a real-world example provided by Montgomery [27] is used to demonstrate the effect of measurement errors with linearly increasing variance on joint monitoring

Table 7 ARLs when $n = 10$ and $\lambda = 0.2$ under RSS approach

UCL (δ, ψ)	1.9886 (C,D)	1.9889	1.9894	1.9926	1.9932
	(0,0)	(0.1)	(0,2)	(1,1)	(3,1)
(0,1)	200.19	199.291	199.729	200.220	200.4830
(0.25,1)	7.648	68.013	87.115	78.3720	100.901
(0.5,1)	2.21	11.095	21.644	13.0760	19.33
(0.75,1)	1.202	4.842	8.241	5.6370	7.35
(1,1)	1.007	2.992	4.767	3.349	4.208
(1.25,1)	1.000	2.096	3.355	2.333	2.956
(0,0.25)	2.000	37.015	104.990	49.421	74.365
(0,0.5)	2.551	55.345	134.726	75.140	102.830
(0,1.1)	18.33	130.545	162.998	144.9170	162.012
(0,1.2)	6.254	73.729	129.951	81.1440	108.527
(0,1.3)	3.694	43.721	87.907	53.6500	75.486
(0.25,0.25)	2.000	57.026	162.803	80.078	108.526
(0.25,0.5)	2.867	76.732	179.923	97.904	134.361
(0.25,1.1)	6.328	45.515	70.451	57.9400	78.262
(0.25,1.2)	4.474	33.321	55.938	39.6080	60.188
(0.5,0.25)	2.018	12.410	27.642	15.473	22.522
(0.5,0.5)	2.110	12.513	28.637	15.923	23.252
(0.5,1.1)	2.154	10.313	19.945	12.7900	18.012
(0.5,1.2)	2.039	9.531	17.436	11.7940	16.519
(0.75,0.25)	1.000	5.031	9.170	5.886	7.658
(0.75,0.5)	1.051	5.060	9.236	5.968	7.736
(0.75,1.1)	1.202	4.761	8.343	5.5010	7.036
(0.75,1.2)	1.245	4.669	7.633	5.4130	6.999
(1,0.25)	1.000	2.91	5.069	3.410	4.211
(1,0.5)	1.000	3.001	5.077	3.494	4.277
(1,1.1)	1.012	2.933	4.837	3.349	4.133
(1,1.2)	1.009	2.921	4.665	3.271	4.029
(1.25,0.25)	1.000	2.062	3.344	2.413	2.878
(1.25,0.5)	1.000	2.092	3.413	2.414	2.924
(1.25,1.1)	1.001	2.128	3.305	2.367	2.878
(1.25,1.2)	1.000	2.094	3.207	2.323	2.872

of process mean and variability. In this dataset, 25 subgroups containing five observations, as given in the second to sixth columns in Table 8, were obtained in which the inside diameter of forged automobile engine piston rings in millimeters (mm) is measured. This dataset is also used by Ghashghaei et al. [28]. They used three normality tests including Anderson–Darling, Rayan–Joiner and Kolmogorov–Smirnov, to find out that the dataset follows a normal distribution with the mean and the standard deviation of 74 and 0.01, respectively. Here through simulation experiments, we set the UCL value equal to 2.8728 to have $ARL_0 \approx 200$. First, the chart statistic corresponding to each sample using error-free data are calculated and plotted. For this purpose, firstly the EWMA and EWMS statistics are calculated by Eqs. (1) and (2) in which $n = 5$, $Z_0 = 74$, $S_0^2 = 0.01$ and $\lambda = 0.2$. Then, by substituting Z_t and S_t^2 in Eqs. (6) and (7), U_t and V_t statistics are obtained. Finally, the maximum of the absolute values of U_t and V_t is considered as the MAX-EWMAMS statistic at sample point t. In the following, the chart statistic under measurement errors are calculated by selecting $A = 0, B = 1, C = 1, D = 1$. It is similar to the error-free data case with this difference that Z_t, S_t^2, U_t, V_t and M_t are substituted by Z', $S_t'^2$, U', V' and M' according to Eqs. (11–16). Figure 4 depicts the statistics in both scenarios, i.e., the error-free and under measurement errors cases. As seen, there is not any false alarm when the measurements are precise. However, a false alarm is received at the tenth sample when the measurements are not error-free.

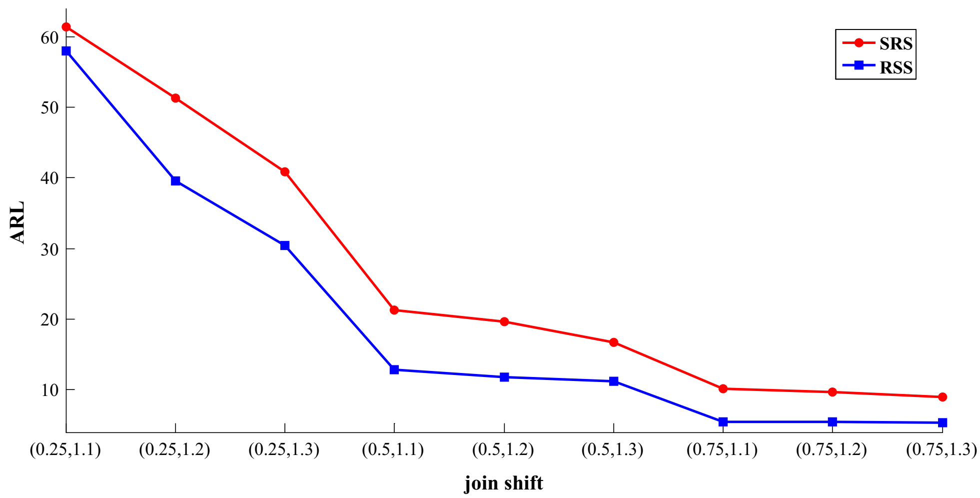

Fig. 3 ARL comparisons between RSS and SRS under simultaneous shifts when $C = 1$, $D = 1$, $\lambda = 0.2$

Table 8 Statistic values for both cases of with and without error

Sample	Observations					Without error			With error		
	1	2	3	4	5	U	V	M	U'	V'	M'
1	74.030	74.002	74.019	73.992	74.008	2.140	1.676	2.140	0.892	0.133	0.892
2	73.995	73.992	74.001	74.011	74.004	1.225	0.984	1.225	0.829	0.141	0.829
3	73.988	74.024	74.021	74.005	74.002	2.007	1.994	2.007	0.071	0.185	0.185
4	74.002	73.996	73.993	74.015	74.009	1.793	1.477	1.793	1.174	0.285	1.174
5	73.992	74.007	74.015	73.989	74.014	1.717	1.539	1.717	2.418	0.663	2.418
6	74.009	73.994	73.997	73.985	73.993	0.517	1.300	1.300	2.291	0.152	2.291
7	73.995	74.006	73.994	74.000	74.005	0.233	0.460	0.460	2.455	0.972	2.455
8	73.985	74.003	73.993	74.015	73.988	0.451	0.891	0.891	2.033	1.140	2.033
9	74.008	73.995	74.009	74.005	74.004	0.072	0.177	0.177	2.223	0.509	2.223
10	73.998	74.000	73.990	74.007	73.995	0.401	0.355	0.401	3.098	1.148	3.098
11	73.994	73.998	73.994	73.995	73.990	1.323	0.647	1.323	2.529	0.677	2.529
12	74.004	74.000	74.007	74.000	73.996	1.028	1.421	1.421	0.703	1.932	1.932
13	73.983	74.002	73.998	73.997	74.012	1.222	1.007	1.222	0.803	1.994	1.994
14	74.006	73.967	73.994	74.000	73.984	2.549	1.486	2.549	0.170	1.539	1.539
15	74.012	74.014	73.998	73.999	74.007	1.352	1.013	1.352	0.228	1.139	1.139
16	74.000	73.984	74.005	73.998	73.996	1.738	0.653	1.738	0.564	0.719	0.719
17	73.994	74.012	73.986	74.005	74.007	1.448	0.551	1.448	0.571	0.544	0.571
18	74.006	74.010	74.018	74.003	74.000	0.273	0.346	0.346	0.916	2.002	2.002
19	73.984	74.002	74.003	74.005	73.997	0.646	0.059	0.646	1.204	1.252	1.252
20	74.000	74.010	74.013	74.020	74.003	0.624	0.341	0.624	0.507	0.987	0.987
21	73.9820	74.001	74.015	74.005	73.996	0.299	0.611	0.611	0.427	1.292	1.292
22	74.004	73.999	73.990	74.006	74.009	0.296	0.048	0.296	0.916	0.943	0.943
23	74.010	73.989	73.990	74.009	74.014	0.408	0.336	0.408	0.374	0.431	0.431
24	74.015	74.008	73.993	74.000	74.010	0.897	0.160	0.897	0.019	0.639	0.639
25	73.982	73.984	73.995	74.017	74.013	0.290	1.407	1.407	0.939	0.118	0.939

Now, to illustrate the importance of simultaneous monitoring of the process mean and variability, the performance of the proposed MAX-EWMAMS control chart is compared to the EWMA chart. In this regard, the EWMA and MAX-EWMAMS chart statistics are plotted under the shift magnitudes of $\delta = 0.75$ and $\psi = 1.2$ in Figs. 5 and 6, respec-

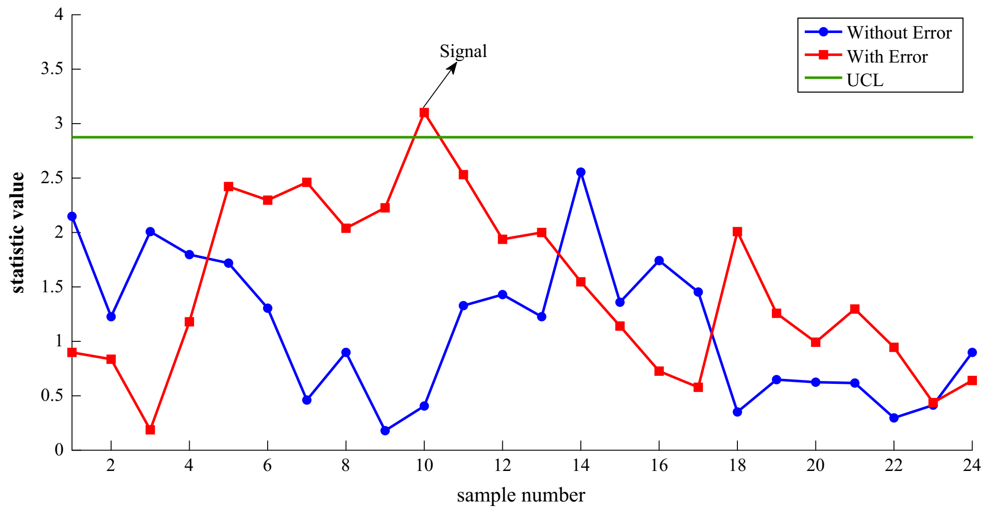

Fig. 4 Effect of measurement errors on the rate of false alarm

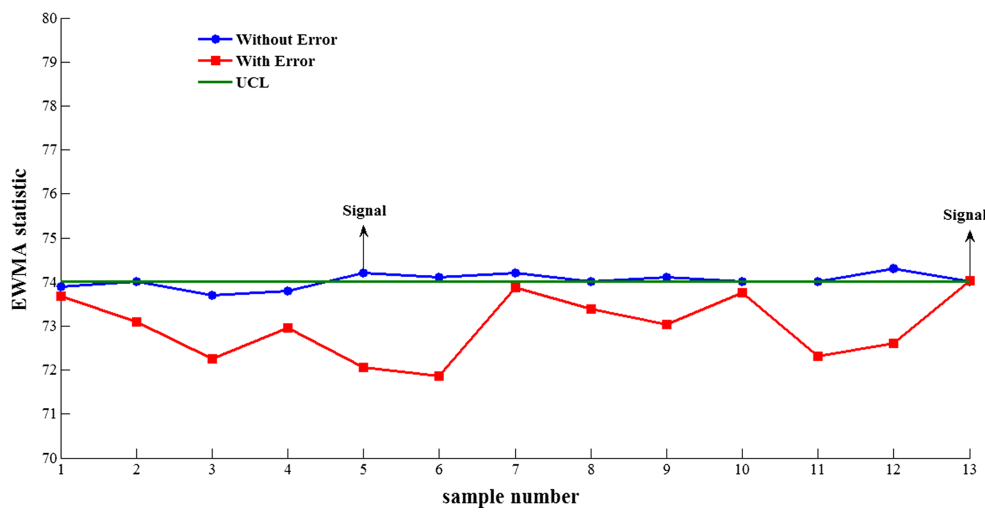

Fig. 5 EWMA statistic values under the shift magnitude of $\delta = 0.75$, $\psi = 1.2$ when $C = 1$, $D = 1$, $\lambda = 0.2$

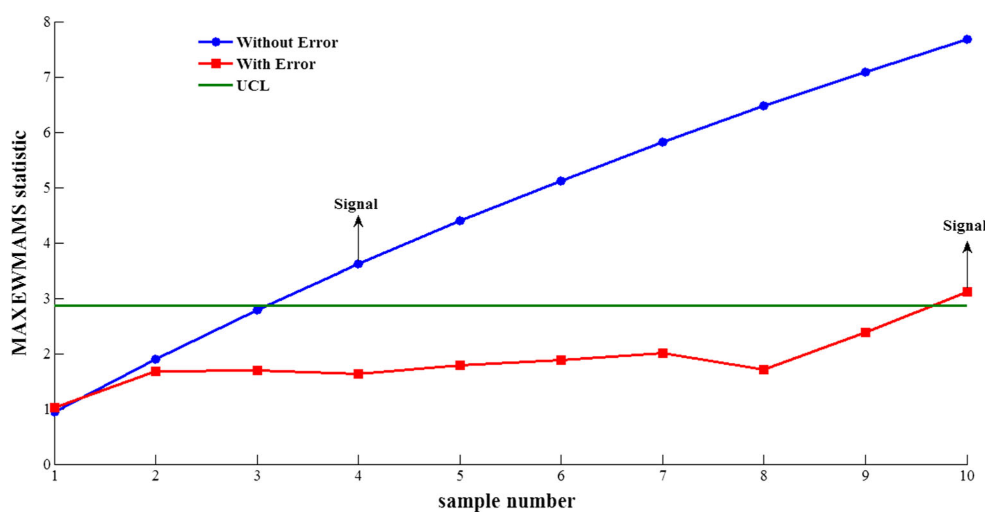

Fig. 6 MAXEWMAS statistic values under the shift magnitudes of $\delta = 0.75$, $\psi = 1.2$ when $C = 1$, $D = 1$, $\lambda = 0.2$

tively, when $C = 1$, $D = 1$ and $\lambda = 0.2$. As can be seen in Figs. 5 and 6, the MAX-EWMAMS chart issues a signal at the fourth and tenth samples while the EWMA chart detects the fault at the 5th and 13th samples in the without and with error data cases, respectively. In other words, the obtained results support this claim that the MAX-EWMAMS chart issues an out-of-control signal earlier than the EWMA in both cases.

8 Conclusions

In this paper, the effect of measurement errors on the performance of one of the most commonly used control charts for simultaneous monitoring of process mean and variability, i.e., the MAX-EWMAMS control chart, was first investigated. An additive covariate model in Phase II analysis was considered in which the assumption of constant variance for measurement error is relaxed. Through simulation studies, we showed that the measurement errors seriously affect the detecting and diagnosing capability of the MAX-EWMAMS control chart. Then, two remedial approaches including the rank set sampling (RSS) and using a larger sample size were extended to compensate for the error effects. The results showed that the performances of both remedial measures were satisfactory. Finally, the effect of measurement errors with linearly increasing variance on the rate of false alarm was evaluated by a real-data example. Investigating the effect of the measurement errors on artificial neural network (ANN)-based control charts is recommended as a future study.

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