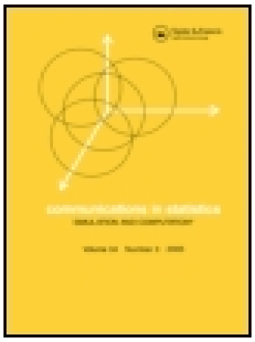

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Phase II monitoring of binary profiles in the presence of within-profile autocorrelation based on Markov Model

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ABSTRACT
This paper introduces a Markov model in Phase II profile monitoring with autocorrelated binary response variable. In the proposed approach, a logistic regression model is extended to describe the within-profile autocorrelation. The likelihood function is constructed and then a particle swarm optimization algorithm (PSO) is tuned and utilized to estimate the model parameters. Furthermore, two control charts are extended in which the covariance matrix is derived based on the Fisher information matrix. Simulation studies are conducted to evaluate the detecting capability of the proposed control charts. A numerical example is also given to illustrate the application of the proposed method.

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1. Introduction

In many practical situations, the quality practitioners are involved in monitoring the quality of a process or product which is characterized by a profile. A profile is referred as a functional relationship between a response variable and one or more explanatory variables. Different control schemes have been proposed by researchers for statistically monitoring of profiles. However, most researches in profile monitoring assumed that the response variable follows Normal distribution. In one of the most important researches, Kang and Albin (2000) proposed two approaches for monitoring the simple linear profiles in a semiconductor manufacturing process. In the first approach, they used multivariate T^2 control chart for monitoring the slope and intercept parameters while in the second approach, they monitored the average and standard deviation of residuals using exponentially weighted moving average (EWMA) and R control charts, respectively. In order to obtain uncorrelated model parameters, Kim et al. (2003) transformed the values of explanatory variable (X) so that the average of the coded X-values changes to zero. Then, based on the transformed X-values, they combined three EWMA-based control charts for simultaneous monitoring of the intercept, the slope as well as the error variance in a simple linear model in Phase II. Some other researchers such as Zou et al. (2006), Zou et al. (2007), Mahmoud (2008), Saghaei et al. (2009) and Zhang et al. (2009) have investigated Phase I and Phase II monitoring of linear profiles. Woodall (2007) also provided a literature review of profile monitoring and suggested more works in this area due to the widespread applications of profile in various sciences.

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In many manufacturing or nonmanufacturing applications, the normality assumption of the response variable of interest is violated. In other words, in some industrial and service environments, the response variable is discrete or categorical such as binary or poisson distributions. In particular, a binary response variable can be used for classifying the outcome of the process to acceptable or non-acceptable products. For instance, consider a cardiac surgical center in which the outcome of the process is defined as the rate of the patient's post-surgical mortality. Obviously in this case, the response variable follows a binary distribution. As a consequence, utilizing a simple linear regression for modeling the relationship between a categorical response variable and one or more independent explanatory variables leads to misleading results. Generally, the profile function can be reasonably handled by a generalized linear model (GLM) when the response variable comes from the distributions of exponential family, including binary, binomial, poisson, exponential, gamma and so on. Recently, there has been a growing interest in GLM profile monitoring area. However, to the best of our knowledge in comparison with the profile monitoring under normal response variables, the number of researches devoted to monitoring GLM profiles are negligible. The GLM profile monitoring approaches are addressed as follows:

Yeh et al. (2009) proposed several Hotelling's T^2 control charts for monitoring logistic profiles in Phase I. They provided a simulation study and compared the performances of the proposed control charts in terms of signal probability criterion by considering the presence of outliers, step shifts and drifts. Shang et al. (2011) combined the EWMA scheme and likelihood ratio test (LRT) to construct a control chart for monitoring a logistic regression model over the time. Their proposed control chart can simultaneously monitor the regression parameters and detects mean shifts in explanatory variables. Izadbakhsh et al. (2011) proposed three methods based on ordinal logistic regression (OLR) for monitoring a profile with the ordinal response variable. Saghaei et al. (2012) introduced two methods for monitoring the logistic regression profiles in Phase II. In the first method, they used the combination of two EWMA-based control charts for mean and variance monitoring of the residuals defined in the logistic regression models while in the second method, they used a multivariate Hotelling's T^2 control chart to monitor the model parameters. Soleymanian et al (2013)introduced four control charts including Hotelling's T^2, multivariate exponentially weighted moving average (MEWMA), LRT and LRT/EWMA for monitoring binary profiles in Phase II. They provided a simulation study and compared the performance of the proposed control charts in terms of ARL criterion. Noorossana and Izadbakhsh (2013) used multinomial logistic regression and attempted to monitor profiles with multinomial response variable. Noorossana et al. (2014) used logistic regression to model nominal responses and proposed three approaches including LRT, MEWMA and support vector machine (SVM) to monitor the quality of the process in Phase II. Shadman et al. (2014) introduced a unified procedure for monitoring the profiles in Phase I such that the response variable belongs to a large class of variables, including continuous, count, or categorical variables. They compared their proposed procedure with the existing control charts under two special cases of binomial and Poisson profiles. Shadman et al. (2017) adopted a change point approach namely Rao score test (RST) for monitoring generalized linear profiles in Phase II. Amiri et al. (2015) extended three control schemes including a T^2-based control chart, LRT method as well as F method for monitoring GLM regression profiles in Phase I. Through a simulation study, they showed that the LRT control chart outperforms two other control charts. Panza and Vargas (2016) studied Phase II monitoring of profiles with Weibull response variable based on the relative log-likelihood ratio statistic.

Most profile monitoring works performed so far including those where mentioned above have assumed that the response values within each profile are independent. However, in many conditions, the measurements are gathered at short time intervals and therefore the observations within each profile are autocorrelated. Also, some sources of variation such as user error, variation of input materials and so on produce autocorrelated observations. It is proved by researchers that the autocorrelation can significantly alter the statistical performance of different profile monitoring approaches. In recent years, the effect of autocorrelation on profile monitoring approaches is addressed by several researchers.

Jensen et al. (2008) suggested linear mixed models for Phase I monitoring of linear profiles in the case of within-profile autocorrelation. They showed via simulation that using linear mixed model approach is preferable to an approach that ignores the autocorrelation structure. Noorossana et al. (2008) explored the effect of neglecting autocorrelation between profiles. They proposed three time series based control charts to eliminate the effect of autocorrelation. Soleimani et al. (2009) studied Phase II monitoring of simple linear profiles when there is a first order autoregressive model, i.e. AR(1) between the observations within each profile. They presented a transformation on the response variable to remove the effect of autocorrelation on the estimates of regression parameters. Then, they investigated the performance of four control charts to monitor the simple linear profiles. Amiri et al. (2010) presented a case study for a profile that can be expressed by a polynomial model. They showed that there is autocorrelation within each profile and therefore using an ordinary least-square method that ignores the autocorrelation is inappropriate. As an alternative approach, they also proposed a method using linear mixed model in Phase I. Soleimani et al. (2013) investigated Phase II monitoring of multivariate simple linear profiles in the presence of within-profile autocorrelation. As a remedial approach, they modified the transformation method by Soleimani et al. (2009) in order to eliminate the effect of autocorrelation on the regression estimates. Keramatpour et al. (2013) suggested a remedial measure to eliminate the effect of between-profiles autocorrelation in Phase II monitoring of polynomial profiles. Afterwards, a control chart based on the generalized linear test (GLT) was proposed to monitor the coefficients of polynomial profiles and an R control chart to monitor the error variance. They also proposed an estimator based on the likelihood ratio approach for the change point estimation of parameters in autocorrelated polynomial profiles. Soleimani et al. (2013) applied the linear mixed models for Phase II monitoring of linear profiles in the presence of within-profile autocorrelation. Then, they evaluated and compared the performance of three extended control charts including Hotteling T^2, MEWMA control chart and multivariate cumulative sum (MCUSUM) control chart. Abdel-Salam et al. (2013) proposed a semiparametric procedure namely a mixed model robust profile monitoring (MMRPM) considering within-profile autocorrelation. Narvand et al. (2013) utilized the linear mixed models to account for autocorrelation of response values within each profile. Then, they used Hotelling's T^2, MEWMA and MCUSUM control charts for Phase II monitoring of the process. Koosha and Amiri (2013) studied the effect of autocorrelation and proposed two remedies to account for the autocorrelation within logistic profiles. Zhang et al. (2014) used the Gaussian process model in order to monitor the linear profiles in Phase II when within-profile data are correlated. They proposed two Shewhart-type multivariate control charts for monitoring the linear trend and the within-profile autocorrelation, separately. Soleimani and Noorossana (2014) studied monitoring of multivariate simple linear profiles in the presence of between-profile autocorrelation. They proposed three time series based methods for eliminating the adverse effect of between-profile autocorrelation on monitoring performance of their control schemes.

Keramatpouret al. (2014) studied Phase II polynomial profile monitoring when there is an AR(1) structure between the error terms in each profile. They employed three control charts based on a remedial measure which is proposed for eliminating the effect of autocorrelation. Khedmati and Niaki (2016) studied Phase II monitoring of general linear profiles in situations with between-profile autocorrelation of error terms. First, they proposed an approach based on the U statistic for eliminating the effect of between-profile autocorrelation. Then, in order to monitor the parameters of the model, they extended a control chart based on the adjusted parameter estimates.

To review the literature of monitoring GLM profiles as well as monitoring autocorrelated profiles with normal response variable systematically, a state-of-the-art survey is depicted and provided in Table A-1 in Appendix A. As seen in Table A-1, none of the referred research works in the context of profile monitoring takes into account the presence of autocorrelation within-profile in monitoring GLM profiles. Although, in real GLM profile monitoring applications, the consecutive response values within each profile may be correlated.

To illustrate the motivation of our work, consider the manufacturing process of electrolytic capacitors in which the measurements are collected at short time intervals. The electrolytic capacitors are inspected by sampling and the result will either be “pass” or “fail.” The explanatory variables are the type of raw material, level of voltage, frequency and temperature. In this example, the response variable follows a binary distribution. Hence, the relationship between the response variable and the explanatory variables is modeled by a logistic regression model. However, this model is strongly depends on the independency assumption of response variable in different levels of explanatory variables. As mentioned in the motivation example, due to the short interval between measuring the response variables within each profile, the independency assumption of response variables within each profile is violated. In order to deal with this issue and to fill the mentioned research gap, in this paper we focus on Phase II monitoring of binary profiles in which the response values within each profile are correlated. The proposed monitoring schemes not only monitors step shifts in the regression parameters, but also it is capable to detect changes in the autocorrelation coefficient. The differences between our study and other works can be observed in the last row of Table A-1 in Appendix A.

The rest of this paper is organized as follows: The framework of the proposed method is presented in Section 2. Then, the stages of our novel approach for Phase II monitoring of autocorrelated binary profiles is illustrated in Section 3. Simulation studies are given in Section 4 to study the performance of the proposed method in detecting various out-of-control scenarios in terms of ARL criterion. An illustrative example is presented in Section 5 to illustrate the application of the proposed method. Conclusion remarks and a recommendation for future research are given in Section 6.

2. The framework of the proposed methodology

In this section, the proposed methodology for Phase II monitoring of binary logistic profiles in the case of within-profile autocorrelation is illustrated. Our framework methodology consists of five steps as follows:

In step 1, a logistic regression model which takes into account within-profile autocorrelation based on Markov model is developed. Based on the autocorrelated response values in step 1, the likelihood function for each profile is constructed in step 2. In step 3, first a parameter tuning approach based on design of experiments (DOE), desirability function analysis (DFA) and TOPSIS method is presented in order to obtain the optimal parameters of the PSO algorithm. Then, the PSO algorithm with tuned parameters is utilized for estimating

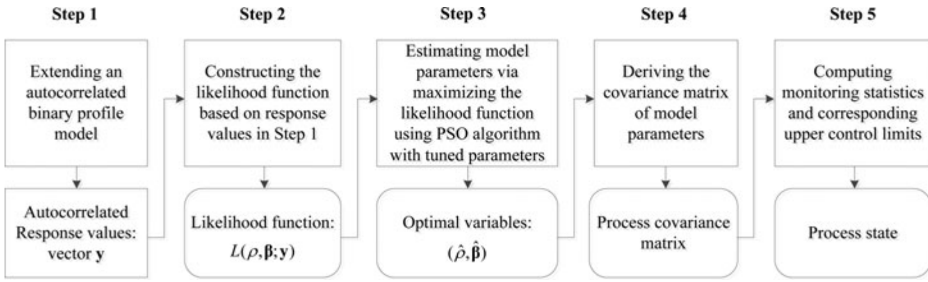

Figure 1. Proposed methodology.

the parameters of the extended logistic regression model. Afterwards in step 4, the covariance matrix considering the autocorrelation structure between consecutive response values in each profile is derived. Finally in step 5, the Hotelling's T^2 and MEWMA statistics based on the estimated model parameters and the covariance matrix is extended and utilized for detecting different step shifts. The proposed methodology is depicted in Figure 1:

3. Steps of the proposed methodology

In order to monitor binary profiles considering within-profile autocorrelation the following steps are applied:

Step 1: Extending an autocorrelated binary profile model

Let y_i be the response value at ith; $i = 1, \dots, n$ experimental setting of a given profile. It is assumed that y_i follows binary distribution with a success probability of π_i. The logistic regression is typically employed to demonstrate a functional relationship between the response and explanatory variables. Thus, under independency assumption on $y_1, y_2, \dots, y_n$ the model representation is:

$$\log it\,(\pi_i) = \log\left(\frac{\pi_i}{1-\pi_i}\right) = e^{\mathbf{x}_i\boldsymbol{\beta}}, \tag{1}$$

where $\mathbf{x_i} = (x_{i1}, x_{i2}, \dots, x_{ip})$ is the explanatory variables corresponding to ith experimental setting, $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_p)^T$ is the logistic regression parameters and $\mathbf{x_i}\boldsymbol{\beta} = \sum_{j=1}^{p} x_{ij}\beta_j$. Equivalently, Eq. (1) can be rewritten as follows:

$$p(y_i = 1) = \pi_i = \frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{1 + e^{\mathbf{X_i}\boldsymbol{\beta}}}. \tag{2}$$

Note that in the logistic regression models, the value of x_{i1} is usually considered equal to one. As the result, β_1 will be the intercept parameter of the model. The success probability in each experimental setting only depends on the value of corresponding explanatory variables. As noted in most manufacturing and nonmanufacturing applications, the independency assumption of response variables within each profile is violated. In such situations, the value of y_i; $i = 1, 2, \dots, n$ not only is related to the vector of explanatory variables in the ith treatment ($\mathbf{x_i}$), but also on the value of observed responses in the previous experimental settings, i.e. $(y_{i-1}, y_{i-2}, \dots, y_1)$. As a consequence, using Eq. (2) for formulating the relationship between explanatory variables and the response variable leads to misleading results. To deal with this issue, we extend a binary regression profile considering a first order autoregressive model, i.e. AR(1) between consecutive response values in each profile. In a given profile, the success probability of response variable in the ith experimental setting can be handled by a binary Markov chain of state space {0, 1} and the transition matrix as follows:

$$\mathbf{P_i} = \begin{bmatrix} 1 - p(y_i = 1|y_{i-1} = 0) & p(y_i = 1|y_{i-1} = 0) \\ 1 - p(y_i = 1|y_{i-1} = 1) & p(y_i = 1|y_{i-1} = 1) \end{bmatrix}, \tag{3}$$

where

$$\begin{aligned} p_{y_{i-1}} &= P(y_i = 1|y_{i-1} = j) \\ &= \begin{cases} P(y_i = 1|y_{i-1} = 0) = P_0 = \frac{1}{1+e^{\mathbf{X_i\beta}}}\left(e^{\mathbf{X_i\beta}} - \rho e^{\frac{1}{2}(\mathbf{X_i}+\mathbf{X_{i-1}})\boldsymbol{\beta}}\right) & \mathrm{y}_{i-1} = 0 \\ P(y_i = 1|y_{i-1} = 1) = P_1 = \frac{1}{1+e^{\mathbf{X_i\beta}}}\left(e^{\mathbf{X_i\beta}} + \rho e^{\frac{1}{2}(\mathbf{X_i}-\mathbf{X_{i-1}})\boldsymbol{\beta}}\right) & \mathrm{y}_{i-1} = 1 \end{cases}. \end{aligned} \tag{4}$$

The proof for Eq. (4) is given in Appendix B. Prior to employ the model, it is important to diagnose the autocorrelation structure for the observed binary data. The general cross-product statistic (Getis, 2009) measures the spatial autocorrelation and can be modified for a sequence of dependent binary random variables. In fact, it measures how strong the tendency is for an arbitrary observed value from its previous observation alike than the other observations located far from the given observation.

For a binary sample path, $y_1, \ldots, y_n$, the general cross-product statistic is:

$$C = \sum_i \sum_j W_{ij} Z_{ij}, \tag{5}$$

where $Z_{ij} = (y_i - y_j)^2$ represents the similarity of ith and jth response values while W_{ij} is the measure of the proximity of y_i and y_j as follows:

$$W_{ij} = \begin{cases} 1 & \text{if } \mathrm{y}_i \textit{ and } y_j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

By normalizing C and using the simple central limit theorem, one may construct a test function to check the significance of the autocorrelation structure among the data. Let S_0, S_1 and S_2 are defined as:

$$S_0 = \sum\sum_{i \neq j} W_{ij}, \quad S_1 = \frac{1}{2}\sum\sum_{i \neq j} (W_{ij} + W_{ji})^2, \quad S_2 = \sum_i (W_{i.} + W_{.i})^2. \tag{7}$$

The values of T_0, T_1 and T_2 are computed by substituting Z_{ij}'s instead of W_{ij}'s in Eq. (7). The C statistic approximately follows a normal distribution with mean $E(C) = \frac{S_0 T_0}{n(n-1)}$ and variance

$$Var(C) = \frac{S_1 T_1}{2n(n-1)} + \frac{(S_2 - 2S_1)(T_2 - 2T_1)}{4n(n-1)(n-2)} + \frac{(S_0^2 + S_1 - S_2)(T_0^2 + T_1 - T_2)}{n(n-1)(n-2)(n-3)} - [E(C)]^2. \tag{8}$$

When $Z = \frac{C-E(C)}{\sqrt{Var(C)}} \notin [-Z_{\alpha/2}, Z_{\alpha/2}]$, the existence of the autocorrelation structure is verified at confidence level of $100(1-\alpha)\%$.

Step 2: Constructing the likelihood function based on response values in Step 1

Yeh et al. (2009) studied the GLM profile monitoring in the case of binomial response variable. They mentioned that the joint likelihood function of observations under independency assumption of response values within each profile is expressed as:

$$L(\pi; \mathbf{y}) = \prod_{i=1}^{n} \binom{\pi_i}{y_i} \pi_i^{y_i} (1 - \pi_i)^{m_i - y_i}, \tag{9}$$

where m_i is the number of observations in ith experimental setting, $\pi = (\pi_1, \pi_2, \ldots, \pi_n)^T$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$. Eq. (9) can be rewritten as follows when there is a single observation in each experimental setting:

$$L(\pi; \mathbf{y}) = \prod_{i=1}^{n} \pi_i^{y_i} (1 - \pi_i)^{1-y_i} \tag{10}$$

The likelihood function in Eq. (10) can only be used when the consecutive response values within each profile are independent. In order to provide the joint likelihood function which takes into account the autocorrelation structure between binary response values within each profile, we use the model provided by Azzalini (1994). Once $y_{1,}y_2, \ldots, y_n$ have been observed, the logarithm of likelihood function corresponding to a given profile in terms of the vector $\boldsymbol{\beta}$ and parameter ρ can be written as follows:

$$\log\left(L(\rho, \boldsymbol{\beta}; \mathbf{y})\right) = \sum_{i=1}^{n} \left[y_i \log it\left(p_{y_{i-1}}\right) + \log(1 - p_{y_{i-1}})\right], \tag{11}$$

where $p_{y_{i-1}}$ can be obtained according to Eq. (4) and $\log it\left(p_{y_{i-1}}\right)$ is given by:

$$\log it\left(p_{y_{i-1}}\right) = \log it\left[P(y_i = 1|y_{i-1})\right] = \begin{cases} \log\left[\dfrac{\frac{1}{1+e^{\mathrm{X_i}\beta}}\left(e^{\mathrm{X_i}\beta} - \rho e^{\frac{1}{2}(\mathrm{X_i}+\mathrm{X_{i-1}})\beta}\right)}{1-\frac{1}{1+e^{\mathrm{X_i}\beta}}\left(e^{\mathrm{X_i}\beta} - \rho e^{\frac{1}{2}(\mathrm{X_i}+\mathrm{X_{i-1}})\beta}\right)}\right] & \mathrm{y}_{i-1} = 0 \\ \log\left[\dfrac{\frac{1}{1+e^{\mathrm{X_i}\beta}}\left(e^{\mathrm{X_i}\beta} + \rho e^{\frac{1}{2}(\mathrm{X_i}-\mathrm{X_{i-1}})\beta}\right)}{1-\frac{1}{1+e^{\mathrm{X_i}\beta}}\left(e^{\mathrm{X_i}\beta} + \rho e^{\frac{1}{2}(\mathrm{X_i}-\mathrm{X_{i-1}})\beta}\right)}\right] & \mathrm{y}_{i-1} = 1 \end{cases}. \tag{12}$$

Note that, it is assumed that the explanatory variables are not random and we keep them to be constant form profile to profile. In a given profile, the estimation of the extended model parameters can be obtained by maximizing the likelihood function based on the fixed explanatory variables and autocorrelated binary response variables.

Step 3: Estimating model parameters via maximizing the likelihood function using PSO algorithm with tuned parameters

In the literature of GLM profile monitoring, it is customary to use the iterative weighted least squares (IWLS) estimation method to obtain the MLE of $\boldsymbol{\beta}$ (see Yeh et al. (2009), Shang et al. (2011),Amiri et al. (2015) for detailed information). When the consecutive response values within each profile are correlated, using IWLS method is no longer reasonable because this method is provided under independency assumption of response values. Moreover, in the presence of within-profile autocorrelation, the likelihood function is much more complex in comparison with Eq. (10). Hence, a heuristic or a metaheuristic procedure can be applied to estimate the model parameters. In this paper, the estimations of the extended model parameters are obtained via maximizing the constructed likelihood function via particle swarm optimization algorithm. In order to improve the estimations of the model parameters, a tuning approach for determining the optimal parameters of PSO algorithm is also introduced. The PSO algorithm as well as the tuning parameter approach is discussed in the subsequent subsections.

Step 3.1: PSO algorithm

The PSO algorithm which is extended by Eberhart and Kennedy (1995) is a metaheuristic global search algorithm based on the swarm behavior of particles. A point in the problem space is referred to a particle which is considered as a candidate solution to the optimization problem. In this algorithm, the collective behavior of a population not only depends on the individual behavior of each particle, but also on the interaction between them. Hence, there is a complex nonlinear relationship between the individual and collective behavior in PSO algorithm. Each particle in PSO algorithm initializes with a random position (solution) and seeks the optimal solution such that it can keep track of its position, velocity (change pattern of solution) and fitness. The position and velocity of each particle is adjusted by considering its own experience and social cooperation by its fitness to the environment. As a result, three following factors can affect the behavior of a given particle: (1) its current position, (2) the best position of the particle among its previous positions (personal best), and (3) the best position among all particles in the population (global best). Each particle improves its position based on the current velocity as well as the personal best and the global best. Therefore, the velocity and position of ith particle, $i = 1, \ldots, M$ in the $(k + 1)$th iteration can be expressed by Eqs. (13) and (14), respectively:

$$\nu_{\mathbf{i}}^{\mathbf{k+1}} = w^{k+1}\nu_{\mathbf{i}}^{\mathbf{k}} + c_1\mathbf{r_1}\left(\mathbf{p_i^k} - \mathbf{x_i^k}\right) + c_2\mathbf{r}_2\left(\mathbf{p_g^k} - \mathbf{x_i^k}\right), \tag{13}$$

$$\mathbf{x_i^{k+1}} = \mathbf{x_i^k} + \nu_{\mathbf{i}}^{\mathbf{k+1}}, \tag{14}$$

where c_1 and c_2 are cognitive learning rate and social learning rate, respectively while r_1 and r_2 are random vectors with uniform distribution within the range $[0, 1]$. In Eq. (13), $\mathbf{p_i^k}$ and $\mathbf{p_g^k}$ represent personal best of the ith particle and global best in kth iteration, respectively. Note that, w in $(k + 1)$th iteration is defined as the inertia weight which can be obtained as:

$$w^{k+1} = w_{damp}.w^k, \tag{15}$$

where w_{damp} can be selected within the range $[0, 1]$ which implies that the effect of inertia in consecutive iterations decreases in comparison with $\mathbf{p_i^k}$ and $\mathbf{p_g^k}$. Utilizing the PSO algorithm for estimating the parameters of the extended logistic regression model is depicted in Figure 2.

Step 3.2: Tuning parameter approach

The heuristic and metaheuristic algorithms are strongly influenced by their parameter values. In To solve this issue, a tuning parameter approach based on design of experiments (DOE), desirability function analysis (DFA) and TOPSIS technique is presented for improving the performance of the extended PSO algorithm. Note that, the technique for order of preference by similarity to ideal solution (TOPSIS) is a compensatory multi-criteria decision method to choose the best solution among a set of alternatives which are characterized by multiple criteria. The compensatory methods provide trade-offs between criteria by compensating a poor result in one criterion by a good result in the other one. Based on identifying weights for each criterion as well as the score of each alternative with respect to each criterion, TOPSIS method selects the alternative which is closest one to the positive ideal solution and the farthest one from the negative ideal solution. The positive ideal solution is the alternative with the best level for all criteria considered while the negative ideal solution is the one which had the worst criteria values. For detailed information about this method refer to Hwang and Yoon (1981). The suggested tuning parameter approach not only seeks to improve the estimation of the regression parameters, but also to minimize the algorithm running time. The procedure for implementing the tuning parameter approach is given as:

1. In this stage, we attempt to efficiently determine the optimal level of PSO parameters using the minimum number of experiments. An experiment design is implemented in which the PSO parameters including maximum number of iterations (*maxit*), number of particles in each population (*npop*), cognitive learning rate (c_1), social learning rate

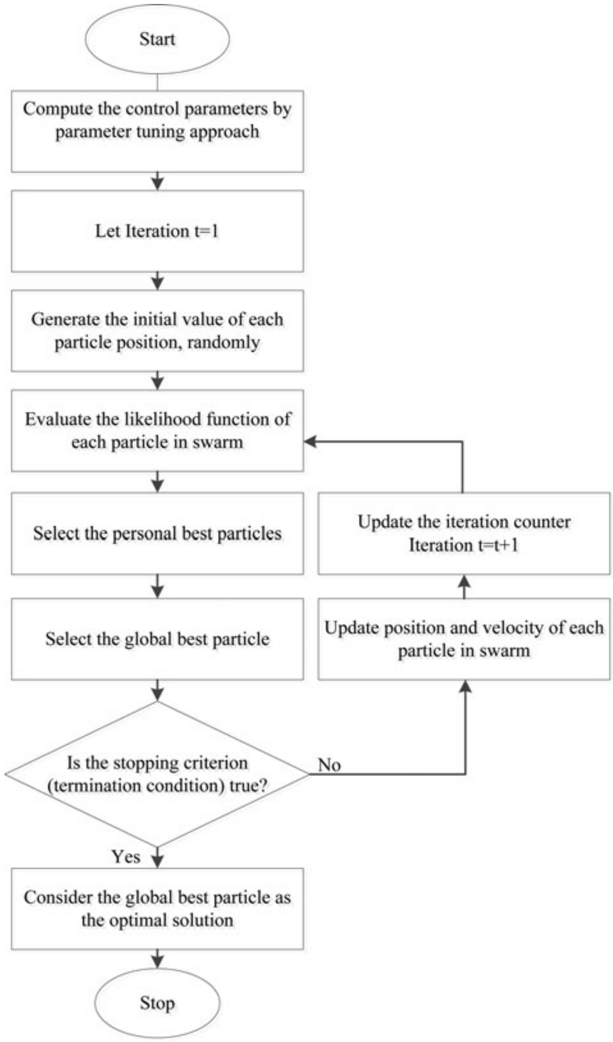

Figure 2. The PSO algorithm to estimate parameters.

(c_2), inertia weight (w) as well as *wdamp* are considered as the controllable factors. The PSO parameters along with their corresponding levels are summarized in Table 1.

Implementing a full factorial design contains totally $2 \times 3 \times 3 \times 3 \times 3 \times 3 = 486$ experiments which is not reasonable due to the time and cost restrictions. Hence, a Taguchi design with orthogonal array $L_{36}(2^1 \times 3^5)$ is created in which the estimated parameters of the extended logistic regression model, i.e. vector $\boldsymbol{\beta}$ and autocorrelation coefficient ρ as well as the computational run time of the algorithm (T) are considered as the response variables.

Note that, A Taguchi design, or an orthogonal array, is a method of designing experiments that usually requires only a fraction of the full factorial combinations. The goal of the Taguchi method is to determine a setting for controllable factors which generates acceptable responses

Table 1. The PSO parameters.

PSO parameter	*maxit*	*npop*	C_1	C_2	*w*	*wdamp*
Number of levels	2	3	3	3	3	3

under natural environmental and process variability. An orthogonal array means the design is balanced so that factor levels are weighted equally. Because of this, each factor can be evaluated independently of all the other factors, so the effect of one factor does not influence the estimation of another factor. In this regard for each treatment, the model parameters are estimated by N replicates and the average value of the estimated model parameters as well as the average computational time of each run are recorded.

2. In this stage, based on the desirability function approach the performance matrix of the L_{36} design is constructed as follows:

$$\mathbf{D} = \begin{bmatrix} d_\rho^1 & d_{\beta_1}^1 & \cdots & d_{\beta_p}^1 & d_T^1 \\ d_\rho^2 & d_{\beta_1}^2 & \cdots & d_{\beta_p}^2 & d_T^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d_\rho^{36} & d_{\beta_1}^{36} & \cdots & d_{\beta_p}^{36} & d_T^{36} \end{bmatrix}, \tag{16}$$

where d_i^j is the desirability function for j^{th} treatment and i^{th} objective function. Note that, the desirability function approach transforms the estimated response variables into a scale-free value by assigning values in the range [0, 1] to the possible value of each response in each treatment. In a given treatment, as the average value of an estimated parameter (β_j; $j = 1, \ldots, p$ or ρ) is closer to its nominal value, the corresponding desirability function will be closer to one. Similarly, in a given treatment, as the average computational time of each run decreases, the corresponding desirability function will be closer to one. The desirability function for a nominal-the-best (NTB), larger the best (LTB) and smaller the best (STB) response variable would be obtained by Eqs. 17–19. The desirability function corresponding to the model parameters ($\beta' s$ and ρ) can be calculated according to Eq. (17) while the desirability function for computational algorithm time (T) can be found in Eq. (19).

$$d_i^j(x) = \begin{cases} \frac{\hat{y}_i(x) - lb_i}{\tau_i - lb_i}; & lb_i \le \hat{y}_i(x) \le \tau_i \\ \frac{\hat{y}_i(x) - ub_i}{\tau_i - ub_i}; & \tau_i \le \hat{y}_i(x) \le ub_i \\ 0; & \hat{y}_i(x) < lb_i \text{ or } \hat{y}_i(x) > ub_i \end{cases}, \tag{17}$$

$$d_i^j(x) = \begin{cases} 0; & \hat{y}_i(x) \le lb_i \\ \frac{\hat{y}_i(x) - lb_i}{ub_i - lb_i}; & lb_i \le \hat{y}_i(x) \le ub_i \\ 1; & \hat{y}_i(x) \ge ub_i \end{cases}, \tag{18}$$

$$d_i^j(x) = \begin{cases} 1; & \hat{y}_i(x) \le lb_i \\ \frac{ub_i - \hat{y}_i(x)}{ub_i - lb_i}; & lb_i \le \hat{y}_i(x) \le ub_i \\ 0; & \hat{y}_i(x) \ge ub_i \end{cases} \tag{19}$$

3. In this step, the performance matrix is normalized as follows:

$$Nd_i^j = \frac{d_i^j}{\sqrt{\sum_{j=1}^{36} (d_i^j)^2}} i = 1, \ldots, p+2,\ j = 1, \ldots, 36, \tag{20}$$

where Nd_i^j is the normalized value of desirability function for jth treatment and ith objective function.

4. In this step, the weighted normalized performance matrix is computed. In this regard, each column of matrix $\mathbf{ND} = [Nd_i^j]_{j,i}$ is multiplied by the relative importance of the

corresponding objective as follows:

$$\mathbf{V} = \begin{bmatrix} W_\rho \times Nd_\rho^1 & W_{\beta_1} \times Nd_{\beta_1}^1 & \cdots & W_{\beta_p} \times Nd_{\beta_p}^1 & W_T \times Nd_T^1 \\ W_\rho \times Nd_\rho^2 & W_{\beta_1} \times Nd_{\beta_1}^2 & \cdots & W_{\beta_p} \times Nd_{\beta_p}^2 & W_T \times Nd_T^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ W_\rho \times Nd_\rho^{36} & W_{\beta_1} \times Nd_{\beta_1}^{36} & \cdots & W_{\beta_p} \times Nd_{\beta_p}^{36} & W_T \times Nd_T^{36} \end{bmatrix}$$
$$= \begin{bmatrix} V_\rho^1 & V_{\beta_1}^1 & \cdots & V_{\beta_p}^1 & V_T^1 \\ V_\rho^2 & V_{\beta_1}^2 & \cdots & V_{\beta_p}^2 & V_T^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ V_\rho^{36} & V_{\beta_1}^{36} & \cdots & V_{\beta_p}^{36} & V_T^{36} \end{bmatrix}, \tag{21}$$

where W_i is the relative importance of the ith objective and $V_i^j = W_i \times Nd_i^j$ is the weighted normalized desirability function for jth treatment and ith objective function.

5. In this step, the vector of positive and negative ideal solutions denoted by $\mathbf{V}^+$ and $\mathbf{V}^-$, respectively are determined as follows:

$$\mathbf{V}^+ = \left((\max_j V_\rho^j, \max_j V_{\beta_1}^j, \ldots, \max_j V_{\beta_p}^j, \max_j V_T^j) | j = 1, \ldots, 36 \right)$$
$$= \left(V_\rho^+, V_{\beta_1}^+, \ldots, V_{\beta_p}^+, V_T^+ \right), \tag{22}$$
$$\mathbf{V}^- = \left((\min_j V_\rho^j, \min_j V_{\beta_1}^j, \ldots, \min_j V_{\beta_p}^j, \min_j V_T^j) | j = 1, \ldots, 36 \right)$$
$$= \left(V_\rho^-, V_{\beta_1}^-, \ldots, V_{\beta_p}^-, V_T^- \right). \tag{23}$$

6. Now, we have to measure the distance of vector $\mathbf{V_j} = (V_\rho^j, V_{\beta_1}^j, \ldots, V_{\beta_p}^j)$ from $\mathbf{V}^+$ and $\mathbf{V}^-$ For this purpose the simple Euclidean distance of j^{th} treatment from the positive and negative ideal solutions is employed as:

$$S^{j^+} = \sqrt{\left(V_\rho^j - V_\rho^+\right)^2 + \left(V_{\beta_1}^j - V_{\beta_1}^+\right)^2 + \ldots + \left(V_{\beta_p}^j - V_{\beta_p}^+\right)^2 + \left(V_T^j - V_T^+\right)^2}, \tag{24}$$

$$S^{j^-} = \sqrt{\left(V_\rho^j - V_\rho^-\right)^2 + \left(V_{\beta_1}^j - V_{\beta_1}^-\right)^2 + \ldots + \left(V_{\beta_p}^j - V_{\beta_p}^-\right)^2 + \left(V_T^j - V_T^-\right)^2}. \tag{25}$$

7. In this step, the relative adjacent of the jth treatment from the positive ideal solution ($rs_j \in [0, 1]$) is computed by Eq. (26). As rs_j tends to the value of 1, the desirability of the jth treatment increases.

$$rs_j = \frac{S_j^-}{S_j^+ + S_j^-}. \tag{26}$$

8. Finally in this step, for each level of parameters, we calculate the average value of relative adjacent in the corresponding treatments.

Step 4: Deriving the covariance matrix of model parameters

The covariance matrix of the parameters of the logistic regression model in the case of independency assumption can be estimated as follows:

$$\mathbf{\Sigma_0} = (\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X})^{-1}, \tag{27}$$

where

$$\mathbf{W} = \begin{pmatrix} \pi_1(1-\pi_1) & 0 & \cdots & 0 \\ 0 & \pi_2(1-\pi_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \pi_n(1-\pi_n) \end{pmatrix} \tag{28}$$

Eq. (28) neglects the autocorrelation structure of response values within each profile. Therefore, using Eq. (27) for calculating the covariance matrix of the estimated parameters in the extended logistic regression model leads to the misleading results. To deal with this issue, we proposed a new approach for deriving the covariance matrix of the estimated model parameters denoted by $\mathbf{\Sigma}_{\hat{\mathrm{U}}}$ where $\hat{\mathbf{U}} = (\hat{\rho}, \hat{\beta}_1, \ldots, \hat{\beta}_p)$ based on the Fisher information matrix approach. Deriving the covariance matrix of the extended model parameters is given in Appendix C.

Step 5: Computing monitoring statistics and corresponding upper control limits

In this step, two control charts including Hotelling's T^2 and MEWMA control charts are proposed for Phase II monitoring of binary profiles in the case of within profile autocorrelation.

Step 5.1: Hotelling's $\mathbf{T}^2$ control chart

The Hotelling's T^2 control chart is used by some researchers for monitoring different profiles. In the case of GLM profiles Yeh et al. (2009), Noorossana and Izadbakhsh (2013), and Amiri et al. (2015) used T^2 control chart in Phase I while some other researchers such as Saghaei et al. (2012), and Soleymanian et al. (2013) utilized this approach in Phase II. The T^2 statistic considering the autocorrelation structure of response values within *j*th profile is modified as follows:

$$T_j^2 = \left(\hat{\mathbf{U}} - E(\hat{\mathbf{U}})\right) \mathbf{\Sigma}_{\hat{\mathrm{U}}}^{-1} \left(\hat{\mathbf{U}} - E(\hat{\mathbf{U}})\right)^T, \tag{29}$$

where $E(\hat{\mathbf{U}}) = (\rho, \beta_1, \beta_2, \ldots, \beta_p)$ is the expected value of the estimated parameters in the extended logistic regression model. Note that $\mathbf{U} = (\rho, \beta_1, \beta_2, \ldots, \beta_p)$ denotes the in-control model parameters and $\mathbf{\Sigma}_{\hat{\mathrm{U}}}$ is the covariance matrix of the estimated model parameters obtained in Step 4. When T_j^2 exceeds the upper control limit (*UCL*), an out-of-control signal will be received. The *UCL* is set such that the in-control average run length (ARL_0) becomes a predetermined value.

Step 5.2: MEWMA control chart

The MEWMA control chart is first proposed by Lowry et al. (1992). In the case of monitoring GLM profiles this approach is first developed by Zou et al. (2007). In the presence of within-profile autocorrelation, the MEWMA statistic for *j*th profile is modified as follows:

$$MEWMA_j = \mathbf{Z_j} \left(\frac{\lambda}{2-\lambda} \mathbf{\Sigma}_{\hat{\mathrm{U}}} \right)^{-1} \mathbf{Z_j}^T, \tag{30}$$

where

$$\mathbf{Z_j} = \lambda \left(\hat{\mathbf{U}} - \mathbf{U}\right) + (1-\lambda)\mathbf{Z_{j-1}}. \tag{31}$$

In Eq. (31) $\mathbf{Z_0} = \mathbf{0}$ and λ is the smoothing parameter satisfying $0 \le \lambda \le 1$. The *UCL* of the MEWMA statistic is obtained through simulation runs such that a desired ARL_0 is achieved.

Table 2. Taguchi design.

treatment	Controllable factors						treatment	Controllable factors					
	max *it*	*npop*	c_1	c_2	w	w_{damp}		max *it*	*npop*	c_1	c_2	w	w_{damp}
1	1	1	1	1	1	1	19	2	1	2	1	3	3
2	1	2	2	2	2	2	20	2	2	3	2	1	1
3	1	3	3	3	3	3	21	2	3	1	3	2	2
4	1	1	1	1	1	2	22	2	1	2	2	3	3
5	1	2	2	2	2	3	23	2	2	3	3	1	1
6	1	3	3	3	3	1	24	2	3	1	1	2	2
7	1	1	1	2	3	1	25	2	1	3	2	1	2
8	1	2	2	3	1	2	26	2	2	1	3	2	3
9	1	3	3	1	2	3	27	2	3	2	1	3	1
10	1	1	1	3	2	1	28	2	1	3	2	2	2
11	1	2	2	1	3	2	29	2	2	1	3	3	3
12	1	3	3	2	1	3	30	2	3	2	1	1	1
13	1	1	2	3	1	3	31	2	1	3	3	3	2
14	1	2	3	1	2	1	32	2	2	1	1	1	3
15	1	3	1	2	3	2	33	2	3	2	2	2	1
16	1	1	2	3	2	1	34	2	1	3	1	2	3
17	1	2	3	1	3	2	35	2	2	1	2	3	1
18	1	3	1	2	1	3	36	2	3	2	3	1	2

4. Performance evaluation

In this section, the performance of the proposed monitoring schemes is evaluated and compared through a numerical example in terms of ARL criterion. Note that ARL is defined as the average number of samples which is taken and plotted until the first sample falls outside the control limits. Without loss of generality, we assume that p is equal to 2. We also assume that when the process is in-control $\mathbf{U} = [\rho, \beta_1, \beta_2]^T = [0.15, 3, 2]^T$. The design matrix of explanatory variables is fixed form profile to profile and considered equal to $\mathbf{X} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \ln(0.02) & \ln(0.04) & \cdots & \ln(0.90) \end{pmatrix}^T$. It is also assumed that for the extended MEWMA control chart λ is equal to 0.2. As mentioned, for tuning the PSO parameters, a Taguchi design was proposed. Table 2 tabulates the Taguchi design with the six controllable factors mentioned in Table 1.

The response values in each row of the proposed design in Table 2 are $Z_1 = \sum_{k=1}^{N} \hat{\rho}_k/N$, $Z_2 = \sum_{k=1}^{N} \hat{\beta}_{1k}/N$, $Z_3 = \sum_{k=1}^{N} \hat{\beta}_{2k}/N$, $Z_4 = \sum_{k=1}^{N} T_k/N$, where N denotes the total number of replicates in each treatment of the Taguchi design and it is considered equal to 1000. Table 3

Table 3. Results of tuning parameters.

PSO parameter	Index of level	Level	Average relative adjacent
max *it*	1	20	0.383
	2	30	0.403
npop	1	500	0.473
	2	1000	0.418
	3	2000	0.289
c_1	1	1	0.466
	2	1.5	0.360
	3	2	0.353
c_2	1	1	0.303
	2	1.5	0.366
	3	2	0.510
w	1	0.5	0.364
	2	0.75	0.399
	3	1	0.416
w_{damp}	1	0.4	0.406
	2	0.6	0.333
	3	0.8	0.440

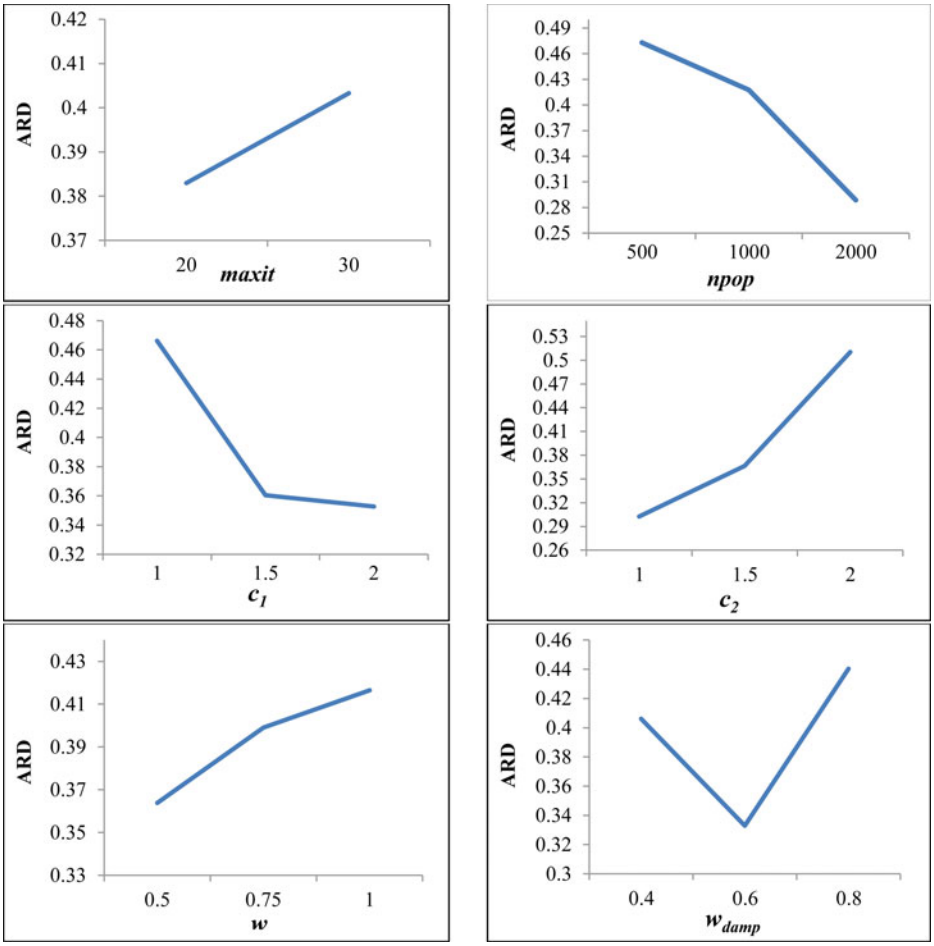

Figure 3. The average of relative adjacent of the levels of the PSO parameters.

contains the average relative adjacent for each level of PSO parameters along with the corresponding levels as well as the index of each level when $W_1 = 0.2$, $W_2 = 0.375$, $W_3 = 0.375$ and $W_4 = 0.05$. Recall that as the average relative adjacent for each level of a given parameter tends to the value of 1, the desirability of the corresponding level increases. The results of tuning parameter are also depicted in Figure 3. The results shows that the optimal PSO parameters are $\max it = 30$, $npop = 500$, $c_1 = 1$, $c_2 = 2$, $w = 1$ and $wdamp = 0.8$.

The covariance matrix of model parameters according to Appendix C is computed equal to $\mathbf{\Sigma}_{\hat{\mathrm{U}}} = \begin{pmatrix} 0.0352 & -0.0899 & -0.0675 \\ -0.0899 & 2.3084 & 1.7293 \\ -0.0675 & 1.7293 & 1.4280 \end{pmatrix}$. For consistency with the literature, we set the UCL for both Hotelling's T^2 and MEWMA control charts by using simulation runs to obtain the probability of Type I error equal to $\alpha = 0.005$ or equivalently the in-control average run length (ARL_0) equal to 200. The UCLs for Hotelling's T^2 and MEWMA control charts through simulation experiments are obtained equal to 11.74 and 10.15, respectively. Then, the performance of two methods to detect different out-of-control scenarios are compared in terms of out-of-control ARL (ARL_1). Note that the step changes in the model parameters are denoted by $\beta_1 + k_1\sigma_{\beta_1}$, $\beta_2 + k_2\sigma_{\beta_2}$ and $\rho + k_3\sigma_\rho$, respectively. The coefficients of k_1, k_2 and k_3 are the magnitude of shift in the intercept, slope and autocorrelation parameters, respectively. Note that, when a given step change in a given model parameter occurs, the in-control value of the parameter

Table 4. The ARLs under different step changes in intercept parameter.

control chart	k_1					
	0	0.2	0.4	0.6	0.8	1
Hotelling's T^2	209.41	124.21	32.72	11.11	4.98	2.76
MEWMA	214.33	22.31	6.73	4.00	2.97	2.55
	k_1					
	1.2	1.4	1.6	1.8	2	2.2
Hotelling's T^2	1.90	1.29	1.17	1.09	1.06	1.03
MEWMA	2.14	1.94	1.83	1.74	1.70	1.46

suddenly changes to an out-of-control value and it does not change until the shift is detected by the control chart.

The performance of both methods in detecting step changes in intercept parameter is displayed in Table 4. As seen in Table 4, the performance of both extended Hotelling's T^2 and MEWMA control charts in detecting different step changes in intercept parameter is satisfactory. For both methods as the magnitude of shift in intercept parameter β_1 increases, the ARL_1 values decrease. As expected, one could see that in small shifts ($k_1 = 0.2, 0.4, 0.6, 0.8, 1$) the MEWMA control chart outperforms the Hotelling's T^2 control chart. While the detecting performance of Hotelling's T^2 control chart under large shifts ($k_1 = 1.2, 1.4, 1.6, 1.8, 2, 2.2$) is better than MEWMA control chart. Note that, the superior performance of MEWMA control chart in detecting small shifts in comparison with Hotelling's T^2 (a Shewhart-type control chart) is due to the memoryless nature of this control chart.

The ARLs of Hotelling's T^2 and MEWMA control charts in detecting the step changes in slope parameter is summarized in Table 5. Similar to Table 4, the results of Table 5 show that the detecting performance of both methods in detecting step change in slope parameter is satisfactory. As we can see in Table 5 when $k_2 = 0.2, 0.4, 0.6, 0.8$, the MEWMA performs better than Hotelling's T^2 control chart. However, the superior performance of Hotelling's T^2 control chart in comparison with MEWMA chart when $k_2 = 1, 1.2, 1.4, 1.6, 1.8, 2, 2.2$ can be concluded form Table 5. Obviously, as the magnitude of shift in slope parameter increases, the ARL_1 values under both methods decrease.

The proposed monitoring schemes not only detect step shifts in the regression parameters, but also are capable to detect changes in autocorrelation coefficient. Table 6 tabulates the performance of both control charts in detecting step shifts in autocorrelation coefficient. As seen both methods can adequately detect various step changes in the autocorrelation coefficient. However, the performance of MEWMA control chart is superior than Hotelling's T^2 control chart under all considered step shifts in autocorrelation coefficient.

Table 5. The ARLs under different step changes in slope parameter.

control chart	k_2					
	0	0.2	0.4	0.6	0.8	1
Hotelling's T^2	209.41	65.09	22.81	7.28	4.40	2.31
MEWMA	214.33	19.36	6.62	3.88	3.00	2.48
	k_2					
	1.2	1.4	1.6	1.8	2	2.2
Hotelling's T^2	1.63	1.24	1.20	1.13	1.03	1.01
MEWMA	2.10	2.00	1.78	1.64	1.66	1.48

Table 6. The ARLs under different step changes in autocorrelation parameter.

	k_3					
control chart	0	0.5	0.75	1	1.25	1.5
Hotelling's T^2	209.41	25.81	18.40	12.89	7.76	5.54
MEWMA	214.33	24.75	16.43	9.97	6.42	4.95

5. An illustrative example

In this section, a numerical example based on the data of Section 4 is presented to illustrate the application of the proposed method. We generate out-of-control autocorrelated profiles according to AR(1) model by inducing a step change of $(k_1 = 0.75, k_2 = 0, k_3 = 0)$ in the vector of model parameters until the chart statistics in both the proposed control charts exceed the *UCL*. The chart statistics for Hotelling's T^2 and MEWMA charts are plotted and illustrated in Figures 4 and 5, respectively. As seen, Hotelling's T^2 triggers an out-of-control signal at 12th sample while MEWMA chart detects the fault at 4th sample. This issue reflects the sensitivity of the extended MEWMA chart to detect small shifts in the model parameters rather than Hotelling's T^2 chart.

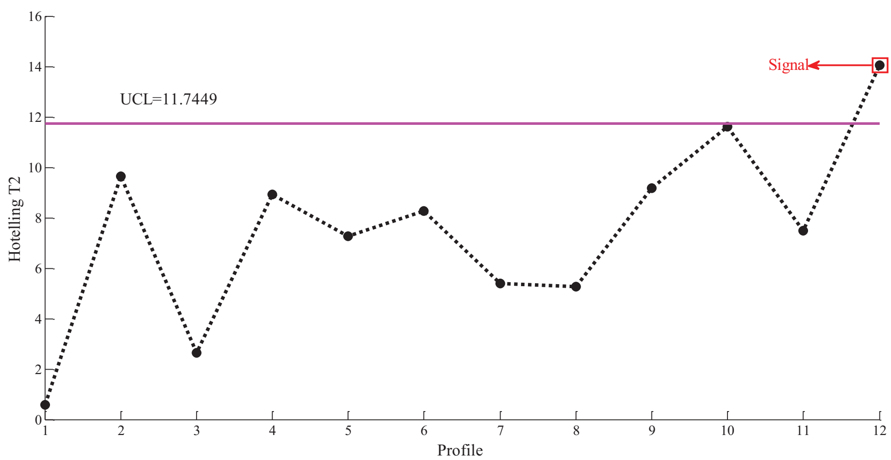

Figure 4. Chart statistics and signal time for T^2 Hotelling's control chart.

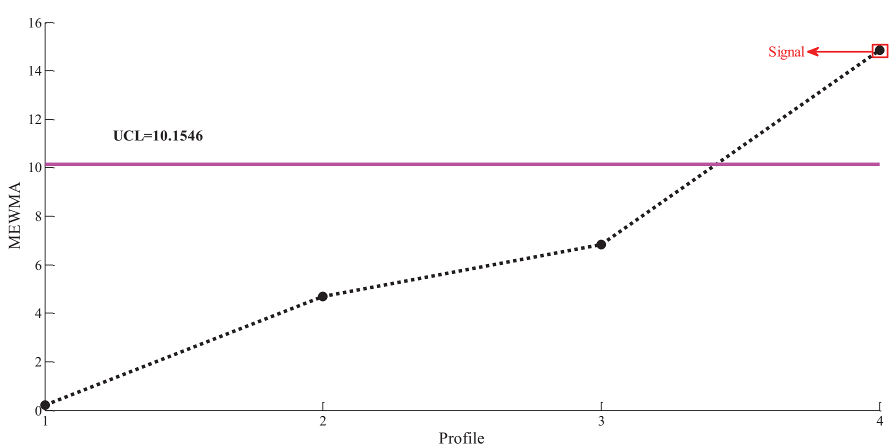

Figure 5. Chart statistics and signal time for MEWMA control chart.

Table 7. The estimated model parameters, the Z values and chart statistics for illustrative example.

subgroup	$\hat{\rho}$	$\hat{\beta}_1$	$\hat{\beta}_2$	Z	T^2	MEWMA
1	0.1866	3.5421	2.1891	− 3.3307	0.5871	0.2113
2	0.0572	3.8903	1.5636	− 2.2425	9.6421	4.7058
3	0.2000	4.4096	2.6194	− 4.0008	2.6431	6.8269
4	0.0755	4.0734	1.7505	− 3.5337	8.9126	**14.8597**
5	0.0670	2.9887	1.0260	− 2.5786	7.2608	—
6	0.1201	4.5000	2.1424	− 3.5337	8.2643	—
7	0.0551	3.7045	1.7121	− 3.7715	5.3828	—
8	0.1948	4.4957	2.3966	− 4.9758	5.2532	—
9	0.0743	4.1534	1.7982	− 5.3271	9.1816	—
10	0.1855	4.2920	1.7786	− 2.8899	11.6203	—
11	0.0869	3.9507	1.7449	− 2.2810	7.4756	—
12	0.1194	4.3480	1.6850	− 2.2415	**14.0431**	—

The estimated model parameters by using the extended PSO algorithm (whose parameters are tuned), the Z values and charts statistics for both methods are also summarized in Table 7. The results confirm the existence of autocorrelation structure at confidence level of 95% for all 12 profiles. Note that, the values of chart statistics of both methods corresponding to signal time are bolded as well in this table.

Note that the proposed methodology to monitor the binary profiles considering the within-profile autocorrelation is coded in MATLAB software and can be easily used by practitioners in real world problems. The codas are available upon request by interested readers and practitioners.

6. Concluding remarks and a future research

Most of research works in profile monitoring area are presented under the assumption that the response variables are independent and identically normally distributed. Some researches on profile monitoring are performed when only one of the normality and independency assumptions is violated. However, in some production systems one can face with monitoring a profile in which both mentioned assumptions are violated, simultaneously. In this paper, we studied monitoring binary profiles in which the response values within each profile are autocorrelated. In the first step of our work, we extended a binary profile model which considers the within autocorrelation structure in each profile. Then, based on the autocorrelated response values, we constructed a likelihood function. In the third step, we estimated the model parameters via maximizing the likelihood function in the second step using PSO algorithm. To improve the efficiency of the PSO algorithm in estimating the model parameters, a tuning parameter approach was also introduced and implemented in the third step. As explained, since the response values within each profile are autocorrelated, using the methods available in literature for computing the covariance matrix of model parameter leads to misleading results. Hence, in the fourth step a new methodology for deriving the covariance matrix of model parameter was proposed. In the final step, two monitoring statistics are developed and utilized for Phase II monitoring of the process. We evaluated the performance of both control charts in detecting step changes in model parameters including regression parameters and autocorrelation coefficient. As explained, one of the superiority of our work rather than the researches in the literature is that it can also detect shifts in the autocorrelation coefficient. The results of simulation study showed that both methods can adequately detect various shifts either in regression parameters as well as autocorrelation coefficient. The results also showed that the MEWMA chart outperforms the Hotelling's T^2 chart in detecting small shifts in regression parameters while the performance of the Hotelling's T^2 chart in detecting large shifts in regression parameters is better than MEWMA chart. In addition, the MEWMA control chart outperforms Hotelling's T^2 control chart in detecting all considered shifts in the autocorrelation coefficient. Finally, we presented an illustrative example to show the application of the proposed method. Monitoring autocorrelated binary profiles in Phase I is recommended as a future research.

Acknowledgment

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Appendix A: A state-of-the-art survey

Table A-1. A review on the literature of monitoring GLM profiles as well as autocorrelated profiles.

	Phase				Type of autocorrelation		
Research	Phase I	Phase II	Relaxing Normality assumption	Relaxing independence assumption	within profile	between profile	monitoring approach
Jensen et al. (2008)	√			√	√		T^2
Noorossana et al. (2008)		√		√		√	EWMA/R, T^2 and EWMA-3
Yeh et al. (2009)	√		√				T^2
Soleimani et al. (2009)		√		√	√		T^2, Residuals-based T^2, EWMA/R and EWMA-3
Amiri et al. (2010)	√			√	√		T^2
Shang et al. (2011)		√	√				EWMA-GLM
Izadbakhsh et al. (2011)		√	√				χ^2, EWMA and $\chi^2/EWMA$
Saghaei et al. (2012)		√	√				EWMA-2 and T^2
Soleymanian et al. (2013)		√	√				T^2, MEWMA, LRT and LRT/EWMA
Noorossana and Izadbakhsh (2013)	√		√				T^2, LRT
Soleimani et al. (2013)		√		√	√		MEWMA and MEWMA/χ^2
Keramatpour et al. (2013)		√		√	√		GLT/R
Soleimani et al. (2013)		√		√	√		T^2, MEWMA and MCUSUM
Abdel-Salam et al. (2013)	√			√	√		T^2
Narvand et al. (2013)		√		√	√		T^2, MEWMA and MCUSUM
Koosha and Amiri (2013)	√		√	√	√		T^2
Noorossana et al. (2014)		√	√				LRT, MEWMA and SVM
Shadman et al. (2014)	√		√				SLRT
Shadman et al. (2017)		√	√				Rao score test
Zhang et al. (2014)		√		√	√		Shewhart-type control charts
Soleimani and Noorossana (2014)		√		√		√	T^2, MEWMA/χ^2 and MEWMA-3
Keramatpour et al. (2014)		√		√	√		T^2, Residuals-based T^2 and EWMA/R
Amiri et al. (2015)	√		√				T^2, LRT and F-test
Panza and Vargas (2016)		√	√				relative log-likelihood ratio statistic based charts
Khedmati and Niaki (2016)		√		√		√	chart based on adjusted parameter estimates
This paper		√	√	√	√		T^2 and MEWMA

Appendix B: Calculating the success probability considering AR(1) model

Azzalini (1994) stated that in a non-stationary case, the marginal mean π_i does not depend either on the past value of the process or on the autocorrelation parameter. To achieve this, he proposed the following model at ith experimental setting:

$$p_j = \pi_i - \rho\left[\pi_i(1-\pi_i)\frac{\pi_{i-1}}{1-\pi_{i-1}}\right]^{\frac{1}{2}} + j\rho[\pi_i(1-\pi_i)]^{\frac{1}{2}}\left[\left(\frac{1-\pi_{i-1}}{\pi_{i-1}}\right)^{\frac{1}{2}} + \left(\frac{\pi_{i-1}}{1-\pi_{i-1}}\right)^{\frac{1}{2}}\right], \tag{B1}$$

where $p_j = P(y_i = 1|y_{i-1} = j)$. If $j = 0$, then

$$p_0 = \frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}} - \rho\left(\frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\frac{\frac{e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}{\frac{1}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}\right)^{\frac{1}{2}} = \frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\left(e^{\mathbf{X_i}\boldsymbol{\beta}} - \rho e^{\frac{1}{2}(\mathbf{X_i}+\mathbf{X_{i-1}})\boldsymbol{\beta}}\right), \tag{B2}$$

Else if $j = 1$, we have

$$\begin{aligned} p_1 = {} & \frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}} - \rho\left(\frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\frac{\frac{e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}{\frac{1}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}\right)^{\frac{1}{2}} \\ & + \rho\left[\frac{e^{\mathbf{X_i}\boldsymbol{\beta}}}{\left(1+e^{\mathbf{X_i}\boldsymbol{\beta}}\right)^2}\right]^{\frac{1}{2}}\left[\left(\frac{\frac{1}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}{\frac{e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}\right)^{\frac{1}{2}} + \left(\frac{\frac{e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}{\frac{1}{1+e^{\mathbf{X_{i-1}}\boldsymbol{\beta}}}}\right)^{\frac{1}{2}}\right]. \end{aligned} \tag{B3}$$

Simplifying Eq. (B3) leads to:

$$\begin{aligned} & \frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\left(e^{\mathbf{X_i}\boldsymbol{\beta}} - \rho e^{\frac{1}{2}(\mathbf{X_i}+\mathbf{X_{i-1}})\boldsymbol{\beta}}\right) + \rho\frac{e^{\mathbf{X_i}\boldsymbol{\beta}/2}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\left[e^{-\mathbf{X_{i-1}}\boldsymbol{\beta}/2} + e^{\mathbf{X_{i-1}}\boldsymbol{\beta}/2}\right] \\ & = \frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\left(e^{\mathbf{X_i}\boldsymbol{\beta}} + \rho e^{\frac{1}{2}(\mathbf{X_i}-\mathbf{X_{i-1}})\boldsymbol{\beta}}\right). \end{aligned} \tag{B4}$$

Appendix C: Deriving the covariance matrix

For simplicity, we consider $p = 2$ and then represent the logarithm of likelihood function $L(\rho, \beta_1, \beta_2)$ by L. The covariance matrix can be obtained by calculating the Hessian matrix as follows:

$$\begin{aligned} H\,(l) &= \left[\frac{\partial^2 l}{\partial\theta_i\partial\theta_j}\right] = \sum_{i=1}^{n} y_i.H\left[\log it\,(p_{y_{i-1}})\right] + \sum_{i=1}^{n} H\left[\log(1-p_{y_{i-1}})\right] \\ &= \begin{bmatrix} \partial^2 l/\partial\rho^2 & \partial^2 l/\partial\rho\partial\beta_1 & \partial^2 l/\partial\rho\partial\beta_2 \\ \partial^2 l/\partial\beta_1\partial\rho & \partial^2 l/\partial{\beta_1}^2 & \partial^2 l/\partial\beta_1\partial\beta_2 \\ \partial^2 l/\partial\beta_2\partial\rho & \partial^2 l/\partial\beta_2\partial\beta_1 & \partial^2 l/\partial{\beta_2}^2 \end{bmatrix}. \end{aligned} \tag{C1}$$

The first element of Eq. (C1) using the chain rule can be obtained as follows:

$$\begin{aligned} H\left[\log it\,(p_{y_{i-1}})\right] &= \left[\frac{\partial^2}{\partial\theta_u\partial\theta_v}\log\left(\frac{p_{y_{i-1}}}{1-p_{y_{i-1}}}\right)\right] \\ &= \left\{\frac{1}{p_{y_{i-1}}(1-p_{y_{i-1}})}\left[\frac{2p_{y_{i-1}}-1}{p_{y_{i-1}}(1-p_{y_{i-1}})}\frac{\partial p_{y_{i-1}}}{\partial\theta_v}\frac{\partial p_{y_{i-1}}}{\partial\theta_u} + \frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_v}\right]\right\}. \end{aligned} \tag{C2}$$

We associate $H[\log it(p_{y_{i-1}})]$ on the value of y_{i-1} by the following notations:

$$H\left[\log it(p_{y_{i-1}})\right] = \begin{cases} H_0\left[\log it(p_{y_{i-1}})\right] & if\ y_{i-1} = 0 \\ H_1\left[\log it(p_{y_{i-1}})\right] & if\ y_{i-1} = 0 \end{cases} \tag{C3}$$

Using chain rule, the second element of Eq. (B1), i.e.$H[\log(1 - p_{y_{i-1}})]$ will be

$$\begin{aligned} &= \left[\frac{\partial^2}{\partial\theta_u\partial\theta_\upsilon}\log\left(1 - p_{y_{i-1}}\right)\right] = \frac{\partial}{\partial\theta_\upsilon}\left(\frac{\partial p_{y_{i-1}}}{\partial\theta_u}\right)\frac{1}{p_{y_{i-1}} - 1} \\ &= \left[\frac{1}{(p_{y_{i-1}} - 1)}\frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_\upsilon} - \frac{1}{\left(p_{y_{i-1}} - 1\right)^2}\left(\frac{\partial p_{y_{i-1}}}{\partial\theta_u}\right)\left(\frac{\partial p_{y_{i-1}}}{\partial\theta_\upsilon}\right)\right]. \end{aligned} \tag{C4}$$

Similar to Eq. (C3), we associate $H[\log(1 - p_{y_{i-1}})]$ on the value of y_{i-1} by the following notations:

$$H\left[\log(1 - p_{y_{i-1}})\right] = \begin{cases} H_0\left[\log(1 - p_{y_{i-1}})\right] & if\ y_{i-1} = 0 \\ H_1\left[\log(1 - p_{y_{i-1}})\right] & if\ y_{i-1} = 0 \end{cases} \tag{C5}$$

The derivations of $H_0[\log it(p_{y_{i-1}})]$ and $H_0[\log(1 - p_{y_{i-1}})]$ are given as follows. Note that if $y_{i-1} = 0$, we have

$$p_{y_{i-1}} = \frac{1}{1 + e^{\mathbf{X_i}\boldsymbol{\beta}}}\left(e^{\mathbf{X_i}\boldsymbol{\beta}} - \rho e^{\frac{1}{2}(\mathbf{X_i}+\mathbf{X_{i-1}})\boldsymbol{\beta}}\right). \tag{C6}$$

Then, if $\theta_u = \theta_\upsilon = \rho$:

$$\frac{\partial p_{y_{i-1}}}{\partial\theta_u} = \frac{\partial p_{y_{i-1}}}{\partial\rho} = \frac{-e^{\frac{1}{2}(\mathbf{X_i}+\mathbf{X_{i-1}})\boldsymbol{\beta}}}{1 + e^{\mathbf{X_i}\boldsymbol{\beta}}},\ \frac{\partial^2 p_{y_{i-1}}}{\partial\theta_i\partial\theta_j} = \frac{\partial^2 p_{y_{i-1}}}{\partial\rho^2} = 0. \tag{C7}$$

If $\theta_u = \rho$ and $\theta_\upsilon = \beta_j;\ j = 1, 2$:

$$\frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_\upsilon} = \frac{-\frac{1}{2}\left(x_i^{(j)} + x_{i-1}^{(j)}\right)e^{\frac{1}{2}(\mathbf{x_i}+\mathbf{x_{i-1}})\boldsymbol{\beta}}(1 + e^{\mathbf{x}_i\boldsymbol{\beta}}) + x_i^{(j)}e^{\mathbf{x}_i\boldsymbol{\beta}}e^{\frac{1}{2}(\mathbf{x_i}+\mathbf{x_{i-1}})\boldsymbol{\beta}}}{\left(1 + e^{\mathbf{x}_i\boldsymbol{\beta}}\right)^2}, \tag{C8}$$

where $\mathbf{x_i} = [x_i^0, x_i^1]$. If $\theta_u = \beta_j;\ j = 1, 2$, we have:

$$\frac{\partial p_{y_{i-1}}}{\partial\beta_j} = \frac{-x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}}}{1 + e^{\mathbf{x_i}\boldsymbol{\beta}}}p_{y_{i-1}} + \frac{1}{1 + e^{\mathbf{x_i}\boldsymbol{\beta}}}\left[x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}} - \frac{\rho}{2}(x_i^{(j)} + x_{i-1}^{(j)})e^{\frac{1}{2}(\mathbf{x_i}+\mathbf{x_{i-1}})\boldsymbol{\beta}}\right]. \tag{C9}$$

It is obvious that when $\theta_u = \beta_j;\ j = 1, 2$ and $\theta_\upsilon = \rho$, $\frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_\upsilon}$ can be obtained according to Equation (C8). In the next step, when $\theta_u = \beta_j$ and $\theta_\upsilon = \beta_k$, we have

$$\begin{aligned} \frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_\upsilon} &= \frac{e^{\mathbf{x_i}\boldsymbol{\beta}}}{1 + e^{\mathbf{x_i}\boldsymbol{\beta}}} \\ &\times \left\{\begin{array}{l} x_i^{(j)}x_i^{(k)} - \frac{\rho}{4}\left(x_i^{(j)} + x_{i-1}^{(j)}\right)\left(x_i^{(k)} + x_{i-1}^{(k)}\right)e^{\frac{1}{2}(\mathbf{x_{i-1}}-\mathbf{x_i})\boldsymbol{\beta}} - \\ \frac{x_i^{(k)}}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}\left[x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}} - \frac{\rho}{2}(x_i^{(j)} + x_{i-1}^{(j)})e^{\frac{1}{2}(\mathbf{x_i}+\mathbf{x_{i-1}})\boldsymbol{\beta}}\right] - x_i^{(j)}x_i^{(k)}\frac{1}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}p_{y_{i-1}} - x_i^{(j)}\frac{\partial p_{y_{i-1}}}{\partial\beta_k} \end{array}\right\}. \end{aligned} \tag{C10}$$

Now, determining the values of $H_1[\log it\,(p_{y_{i-1}})]$ and $H_1[\log(1-p_{y_{i-1}})]$ are discussed as follows. Recall that if $y_{i-1}=1$, we have

$$p_{y_{i-1}}=\frac{1}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}}\left(e^{\mathbf{X_i}\boldsymbol{\beta}}+\rho e^{\frac{1}{2}(\mathbf{X_i}-\mathbf{X_{i-1}})\boldsymbol{\beta}}\right). \tag{C11}$$

If $\theta_u=\theta_v=\rho$, then

$$\frac{\partial p_{y_{i-1}}}{\partial\theta_u}=\frac{\partial p_{y_{i-1}}}{\partial\rho}=\frac{e^{\frac{1}{2}(\mathbf{X_i}-\mathbf{X_{i-1}})\boldsymbol{\beta}}}{1+e^{\mathbf{X_i}\boldsymbol{\beta}}},\ \frac{\partial^2 p_{y_{i-1}}}{\partial\theta_i\partial\theta_j}=\frac{\partial^2 p_{y_{i-1}}}{\partial\rho^2}=0. \tag{C12}$$

Now when $\theta_u=\rho$ and $\theta_v=\beta_j;\ j=1,2$, we have

$$\frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_v}=\frac{\frac{1}{2}\left(x_i^{(j)}-x_{i-1}^{(j)}\right)e^{\frac{1}{2}(\mathbf{x_i}-\mathbf{x_{i-1}})\boldsymbol{\beta}}(1+e^{\mathbf{x}_i\boldsymbol{\beta}})-x_i^{(j)}e^{\mathbf{x}_i\boldsymbol{\beta}}e^{\frac{1}{2}(\mathbf{x_i}-\mathbf{x_{i-1}})\boldsymbol{\beta}}}{\left(1+e^{\mathbf{x}_i\boldsymbol{\beta}}\right)^2}. \tag{C13}$$

Consider $\theta_u=\beta_j;\ h=1,2$, then

$$\frac{\partial p_{y_{i-1}}}{\partial\beta_j}=\frac{-x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}}}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}p_{y_{i-1}}+\frac{1}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}\left[x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}}+\frac{\rho}{2}(x_i^{(j)}-x_{i-1}^{(j)})e^{\frac{1}{2}(\mathbf{x_i}-\mathbf{x_{i-1}})\boldsymbol{\beta}}\right]. \tag{C14}$$

Finally, when $\theta_u=\beta_j;\ j=1,2$ and $\theta_v=\beta_k;\ k=1,2$, the following equation is obtained:

$$\frac{\partial^2 p_{y_{i-1}}}{\partial\theta_u\partial\theta_v}=\frac{e^{\mathbf{x_i}\boldsymbol{\beta}}}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}\times\left\{\begin{array}{l}x_i^{(j)}x_i^{(k)}+\frac{\rho}{4}\left(x_i^{(j)}-x_{i-1}^{(j)}\right)\left(x_i^{(k)}-x_{i-1}^{(k)}\right)e^{-\frac{1}{2}(\mathbf{x_i}+\mathbf{x_{i-1}})\boldsymbol{\beta}}-\\ \frac{x_i^{(k)}}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}\left[x_i^{(j)}e^{\mathbf{x_i}\boldsymbol{\beta}}+\frac{\rho}{2}(x_i^{(j)}-x_{i-1}^{(j)})e^{\frac{1}{2}(\mathbf{x_i}-\mathbf{x_{i-1}})\boldsymbol{\beta}}\right]-x_i^{(j)}x_i^{(k)}\frac{1}{1+e^{\mathbf{x_i}\boldsymbol{\beta}}}p_{y_{t-1}}-x_i^{(j)}\frac{\partial p_{y_{i-1}}}{\partial\beta_k}\end{array}\right\}. \tag{C15}$$

Eq. (C16) expressed the relationship between Hessian matrix of likelihood function and covariance matrix of the estimated parameters:

$$-\boldsymbol{\Sigma}_{\hat{\mathbf{U}}}{}^{-1}=E\left[H(l)\right], \tag{C16}$$

where:

$$\begin{aligned}E\left[H(l)\right]&=E\left[\sum_{i=1}^{n}y_i.H\left[\log it\,(p_{y_{i-1}})\right]+\sum_{i=1}^{n}H\left[\log(1-p_{y_{i-1}})\right]\right]\\&=\sum_{i=1}^{n}E\left[y_i.H\left[\log it\,(p_{y_{i-1}})\right]+H\left[\log(1-p_{y_{i-1}})\right]\right]\\&=\sum_{i=1}^{n}E\left[y_i.H\left[\log it\,(p_{y_{i-1}})\right]\right]+\sum_{i=1}^{n}E\left[H\left[\log(1-p_{y_{i-1}})\right]\right].\end{aligned} \tag{C17}$$

For ith; $i=1,\ldots,n$ experimental setting, the value of $E[y_i.H[\log it\,(p_{y_{i-1}})]]$ is related to the valueof y_{i-1}. Hence

$$\begin{aligned}E\left[y_i.H\left[\log it\,(p_{y_{i-1}})\right]\right]&=E\left\{E\left[y_i.H\left[\log it\,(p_{y_{i-1}})|y_{i-1}\right]\right]\right\}\\&=E\left[H\left[\log it\,(p_{y_{i-1}})\right].E\left(y_i|y_{i-1}\right)\right].\end{aligned} \tag{C18}$$

According to Eq. (4), since $y_i|y_{i-1}$ follows a binary distribution with parameter $p_{y_{i-1}}=P(y_i=1|y_{i-1})$, then we have $E(y_i|y_{i-1})=p_{y_{i-1}}=P(y_i=1|y_{i-1})$. As a consequence, Eq. (C18) can be rewritten as follows:

$$E\left[H\left[\log it\left(p_{y_{i-1}}\right)\right].p_{y_{i-1}}\right] \\ = P(y_{i-1}=1).H_1\left[\log it\left(p_{y_{i-1}}\right)\right].P(y_i=1|y_{i-1}=1) \\ + P(y_{i-1}=0).H_0\left[\log it\left(p_{y_{i-1}}\right)\right].P(y_i=1|y_{i-1}=0), \tag{C19}$$

where $P(y_{i-1}=1)=\frac{e^{\mathbf{x}_{i-1}\boldsymbol{\beta}}}{1+e^{\mathbf{x}_{i-1}\boldsymbol{\beta}}}$ and $P(y_{i-1}=0)=1-\frac{e^{\mathbf{x}_{i-1}\boldsymbol{\beta}}}{1+e^{\mathbf{x}_{i-1}\boldsymbol{\beta}}}$. Similarly

$$E\left[H\left[\log(1-p_{y_{i-1}})\right]\right]=P(y_{i-1}=1).H_1\left[\log(1-p_{y_{i-1}})\right]+P(y_{i-1}=0).H_0\left[\log(1-p_{y_{i-1}})\right] \tag{C20}$$

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