



## Review

## Measurement errors in statistical process monitoring: A literature review



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## ABSTRACT

In most industrial applications, the measures performed on inspected units are often strongly contaminated by either the inspector or the measuring device leading to measurement errors. It is recognized that the measurement errors affect the performance of control charts in various statistical process monitoring applications. In this paper, we present a conceptual classification scheme based on content analysis method to analyze and categorize the researches which have explored the effect of measurement errors on different aspects of statistical process monitoring (SPM). Moreover, based on 60 relevant papers in this field, the research gaps are mentioned and some directions to motivate the future studies are provided.

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## 1. Introduction

Statistical process monitoring (SPM) helps the quality managers and practitioners to improve the quality of the products by reducing the process variability. The control chart introduced by Shewhart in 1924 is viewed as the most important tools of SPM which is usually used for monitoring the process mean or variability. One of the main purposes of the control charts is to distinguish between assignable and common causes of variation. The process that works only in the presence of common causes of variability is said to be statistically in-control. When a given sample falls outside the control limits, the control chart triggers an out-of-control signal. If the issued signal from control chart is not a false alarm, corrective action(s) should be implemented to eliminate the assignable cause(s) and, consequently, return the process to the in-control state.

In real statistical process monitoring applications, two sources of variation including variations due to the manufacturing process and the variations due to the measurement instruments can cause imprecise measurements. Most of research works in statistical process monitoring assumes that the measurements are accurate. However, an exact measurement is a rare phenomenon in any manufacturing and service environment where human involvement is necessary. As a consequence, the existence of errors due to either the measurement instruments and/or operators is inevitable. In other words, a difference between the real quantities and the measured ones will always exist even with highly sophisticated advanced measuring instruments. For instance, in a production line filling bottles, it is impossible to obtain the exact volume of the liquid inside the bottles; in a mass-spectrometry analysis in analytical laboratories, measurement errors usually occur in the generation and measurement of peak area; in medical applications, the measurements of blood pressure by an analog machine may not always give exact readings (Riaz, 2014).

It is stipulated in the literature that due to an increase in the process variability, imprecise measurements affect the performance of different schemes in SPM areas. Such adverse effects can be considered from two general points of view: (1) The measurement errors reduce the performance of monitoring schemes in detecting out-of-control situations and (2) The measurement errors increase the rate of false alarms. As the variance of measurement errors increases, the uncertainty due to the increasing in the process variability will be increased. Hence, the statistical features of control chart to detect the process disturbances will be affected especially when the variability due to the measurement errors is large relative to the process variability. However, in most researches, the effect of measurement errors on the performance of various SPM areas is neglected. Fortunately in recent years, the quality engineering researchers have produced some efforts to investigate the effect of measurement errors on the performance of different SPM areas especially for statistical design of various control charts.

Most of researches have only investigated the effect of measurement errors on the performance of a given SPM procedure while, some other researches, particularly the recent ones have attempted to present remedial approaches to compensate for the effect of measurement errors. Based on a rigorous content analysis method, in this paper we aim to present an overview on the effect of measurement errors on different areas of SPM, and to provide a conceptual classification scheme for articles in this area. For this

purpose, 60 journal papers have been searched and reviewed. The rest of this article is organized as follows: In Section 2, the survey methodology based on two major steps for providing and categorizing the relevant journal papers is discussed. In Section 3, a conceptual classification scheme to analyze the selected papers in Section 2 is presented. For this purpose, the questions and the possible answers from different points of view are described. Categorizing the relevant papers according to four criteria namely (1) the number of documents per year, (2) the title of journal considering the number of relevant publications, (3) the name of author/co-author considering the number of relevant publications and (4) the conceptual classification scheme to assess the relevant papers is presented in Section 4. In Section 5, the results are discussed in detailed. Finally, in Section 6, several research gaps are identified and some recommendations for future studies are provided.

## 2. Survey methodology

In this paper, we present a systematic review based on content analysis to conceptually classify the researches which have explored the effect of measurement errors on different areas of SPM. For more and detailed information about content analysis methods, please refer to Kolbe and Burnett (1991). A survey based on content analysis consists of two major steps: (1) defining the sources and procedures to search for the papers which should be analyzed and (2) determining the instrumental categories for the classification of the selected papers (Hachicha & Ghorbel, 2012). These steps in our survey are discussed as follows:

### 2.1. Step 1: Sources and procedure to search for and select the papers

To provide the related research sources to conduct our survey, only journal papers are selected and assessed. The other sources such as books, MSc/PhD theses, conference papers and so on are not considered in our work. The relevant papers are gathered via computerized search using proper keywords such as “measurement errors”, “gauge error”, “gauge measurement errors”, “imprecise measurements”, “contaminated data”, “imprecise data” and so on. Then, the references and citations of each paper are also investigated to find the previous works. This procedure is continued progressively and the computerized search is narrowed. In this procedure, the new relevant papers are added to our analysis during the completion of the paper. Note that, to search and select the relevant papers, main platforms/publishers such as ScienceDirect, Taylor & Francis, Springer Link, Emerald Insight, JSTOR, Inderscience, John Wiley and so on are considered.

### 2.2. Step 2: Classifying the selected papers

After performing the first step, the related papers are classified in terms of four criteria as follows:

- I. The number of documents considering the year of publication.
- II. The title of journals considering the number of relevant publications.
- III. The name of authors/co-authors considering the number of relevant publications.
- IV. The conceptual classification scheme to assess the relevant papers.

**Table 1**  
List of questions and of possible responses.

---

1. What is the SPM area?
(1.1) statistical design of control charts
(1.2) economic design/economic-statistical design
(1.3) process capability analysis
2. What type of measurement errors model is used?
(2.1) additive model
(2.2) multiplicative model
(2.3) four-component model
(2.4) two-component model
3. What type of variance is assumed for the measurement error term?
(3.1) constant
(3.2) linearly increasing
(3.3) constant & linearly increasing
4. Type of quality characteristic?
(4.1) univariate
(4.2) multivariate
(4.3) attribute
(4.4) profile
(4.5) fuzzy
5. Type of process?
(5.1) ordinary
(5.2) multi-stage
(5.3) autocorrelated
(5.4) multi-stage & autocorrelated
6. What type of remedial approach is used to account for the effect of measurement errors?
(6.1) no remedial approach
(6.2) multiple measurements approach
(6.3) Other approaches (increasing $n$ , RSS-based approaches, adjusting control limit coefficient, adjusting lower confidence bounds and critical values, inverse method, MANOVA-based method, Omitting outliers)
(6.4) multiple measurements & other approaches
7. Type of statistic/index which is used to analyze the quality of the process?
7.1. statistical, economic, economic-statistical design of control charts
7.1.1. Shewhart-type statistic ( $\bar{X} - R, \bar{X} - S, T^2, \dots$ )
7.1.2. memory-based statistic (EWMA, CUSUM, ...)
7.1.3. Shewhart-type & memory-based statistics
7.2. process capability
7.2.1. univariate index ( $C_p, C_{pk}, C_{pm}, \dots$ )
7.2.2. multivariate index ( $MC_p, \dots$ )

---

The conceptual classification scheme which is also conducted by Hachicha and Ghorbel (2012) is depicted in Table 1. As seen in Table 1, to conceptually classify the selected papers in step 1, seven questions for each paper should be replied. These questions and the possible responses for each one are discussed in Section 3

### 3. The questions and possible responses for the conceptual classification scheme

Here, the questions and the possible responses for each one are discussed.

#### 3.1. SPM area

- **Statistical design of control charts:** The main goal of SPM is online assessment of the process to check its consistency over time. The most common tool for online assessing of a given process is the control chart which was first introduced by Shewhart in 1924. Designing control charts on the basis of their statistical performance such as the average run length (ARL) criterion (or any run length bases property) is referred to as statistical design of control charts (Woodall, 1985).
- **Economic/economic-statistical design of control charts:** In statistical design of control charts, the chart parameters namely the sample size ( $n$ ), the sampling frequency ( $h$ ) and the control limit coefficient ( $L$ ) are determined such that the desired values for the power of chart to detect a given shift ( $1 - \beta$ ) and the probability of Type I error ( $\alpha$ ) are obtained. However, designing

the parameters of a control chart based on statistical criteria leads to ignore the economic consequences. Determining the parameters of control charts by considering the economic criteria is called as economic design. It should be noted that, an economic design neglects the statistical properties such as probabilities of Type I and Type II errors. To cover the mentioned issues (to improve the statistical features as well as to minimize the cost), the economic-statistical design of control charts are used in which both statistical and economic features of control charts are considered simultaneously.

- **Process capability analysis:** Determining the statistical ability of a process to achieve measurable results that satisfy established specifications is referred to as process capability analysis. In the other words, the process capability indices show how well a process is able to fulfill the customer expectations and to conform to specification limits.

#### 3.2. The type of measurement errors model

As noted, the relationship between the actual and the observed values of the sampled units are mostly expressed by the following three models as follows:

- **Additive model:** The most commonly used model in the literature to characterize the relationship between the actual and observed values of quality characteristics under investigation is the additive model defined as:

$$Y = A + BX + \varepsilon, \quad (1)$$

where  $X$  is the actual value of the quality characteristic under investigation which is assumed to follow a normal distribution with mean  $\mu_x$  and variance  $\sigma_x^2$  and  $A$  and  $B > 1$  are two constants which are fixed. In Eq. (1),  $\varepsilon$  is the measurement errors term which is assumed to follow a normal distribution with a mean value equal to 0 and a given variance (constant or non-constant) and it is assumed to be independent from  $X$ . The variance of measurement errors term is discussed in Section 3.3.

- **Multiplicative model:** The relationship between the actual and observed quantities under multiplicative model is:

$$Y = X\varepsilon, \quad (2)$$

where  $\varepsilon$  which is multiplied with the original variable is an independent random variable with mean value equal to 1 and a given variance.

- **Four-component measurement errors model:** Li and Huang (2009) proposed this model which contains four types of measurement errors in a multivariate case with  $p$  correlated variables  $\{X_1, \dots, X_p\}$ . The formulation of this model for variable  $X_j$  is expressed as follows:

$$Y_j = b_j + s_j X_j + c_j^T V_j + \varepsilon_j, \quad (3)$$

where

- $b_j$  is the measurement error caused by sensor setup/calibration bias or drift when sensors are used in harsh environments.
- $s_j$  is the measurement sensitivity.
- $c_j$  represents the relationship between observed and actual quantities which also depends on the other variables ( $V_j$ ) where  $V_j \in \{X_1, \dots, X_p\}$  but  $V_j \notin X_j$ .
- $\varepsilon_j \sim N(0, \text{var}(\varepsilon_j))$  denotes the sensor noise.

- **Two-component measurement errors (TCME) model:** This type of error model is defined as follows:

$$Y = A + BXe^\eta + \varepsilon, \quad (4)$$

where  $A$  and  $B$  are the intercept and slope constants,  $\varepsilon$  and  $\eta$  are additive and multiplicative random disturbances, respectively which are independently normally distributed variables with a mean equal to 0 and a given variance.

### 3.3. Variance of measurement error term

- **Constant:** In most researches in the literature, the variance of the measurement errors term,  $\varepsilon$  is assumed to be a constant value namely  $\sigma_\varepsilon^2$ . For example, in an additive covariate model with constant variance for the measurement errors term,  $Y$  will be a normally distributed variable as follows:

$$Y \sim N(A + B\mu_x, B^2\sigma_x^2 + \sigma_\varepsilon^2). \quad (5)$$

Based on Eq. (5), it is obvious that due to the measurement errors term, the variance of  $Y$  will be larger than the variance of  $X$ . Therefore, in the presence of measurement errors, the process variability increases.

- **Linearly increasing:** In some applications, the variance of the measurement errors linearly depends on the process level and, therefore, the constant variance assumption is relaxed. In this case,  $\varepsilon$  is a normally distributed variable with mean equal to 0 and variance  $C + D\mu_x$ . Hence, we have  $Y \sim N(A + B\mu_x, B^2\sigma_x^2 + C + D\mu_x)$ , where  $C$  and  $D$  are two other constants which are fixed.
- **Constant & linearly increasing:** In addition to the researches assuming constant and linearly increasing variance, the effect of measurement errors with both constant and linearly increasing variance are addressed in some papers such as Maravelakis,

Panaretos, and Psarakis (2004) and Haq, Brown, Moltchanova, and Al-Omari (2015).

### 3.4. Type of quality characteristics

We classify the literature of the measurement errors effect on SPM into five groups namely univariate, multivariate, attribute, profile as well as fuzzy.

- **Univariate:** A single quality characteristic which is expressed in a continuous scale such as size, weight, volume, time and so on.
- **Multivariate:** Several correlated quality characteristics which are expressed via a continuous scale.
- **Attribute:** One quality characteristic which is countable and characterized in a discrete scale.
- **Profile:** Sometimes, the quality of a product or a process is summarized by a functional relationship between a response variable and one or more explanatory variables which is referred to as “profile”.
- **Fuzzy:** Quality characteristics which contain some sources of uncertainties due to human judgment, evaluations and decisions and are expressed by fuzzy numbers and/or linguistic variables.

### 3.5. Process type

- **Multi-stage process:** In multi-stage processes, the manufacturing process includes several stages. In such situations, the quality of the current stage is affected by the outcome of the previous stage(s).
- **Autocorrelated process:** In autocorrelated processes, the independence assumption of consecutive sampled points is violated. For instance, in manufacturing or non-manufacturing environments when the measurements are gathered at short time intervals, it is reasonable that the observations become autocorrelated.
- **Multi-stage and autocorrelated process:** In addition to the mentioned categories, there is a single research in the literature where the effect of imprecise measurements caused by measurement errors on multi-stage processes is addressed in which the observation are autocorrelated.
- **Ordinary process:** Other processes which are not classified as multi-stage or autocorrelated processes are considered as ordinary processes.

### 3.6. Type of remedial approach

- **No remedial approach:** As explained, the performance of control charts is significantly affected by the measurement errors. Although, it is important to provide some remedial approaches to decrease the adverse effect of measurement errors on SPM procedures, however, in many researches, using remedial approaches is ignored and only the effect of measurement errors on the performance of SPM procedures is investigated.
- **Multiple measurements approach:** One of the most common remedial approaches to compensate for the effect of contaminated data on SPM procedures is the “multiple measurements” approach which was first introduced by Linna and Woodall (2001). In this approach, several measurements per item of each sample are taken instead of a single measurement and then the average of the measured values for each item is calculated. As a result, the variance of the measurement error component in the multiple measurements approach will be smaller than the one when using a single measurement.
- **Other approaches:** As noted previously, the most commonly used method to improve the performance of SPM procedures in the presence of measurement errors is the multiple measurements approach. However, some other approaches namely

increasing sample size (see Abbasi (2016) for example), RSS-based methods (discussed in Ghashghaei, Bashiri, Amiri, and Maleki (in press) and Haq et al. (2015), adjusting control limit coefficients (see Riaz (2014)), adjusting lower confidence bounds and critical values for process capability analysis area (such as in Pearn and Liao (2005)), inverse method (utilized in Villeta, Rubio, Sebastián, and Sanz (2010)), multivariate analysis of variance (MANOVA)-based method (suggested by Scagliarini (2011)) as well as omitting outliers via IQR-based approaches (see Amiri, Ghashghaei, and Maleki (in press)) are also used in some papers.

- **Multiple measurements & other approaches:** The fourth possible category in this regard is devoted to the researches which contain multiple measurement approach which is used along with the other remedial approaches.

### 3.7. Type of statistic/index used to analyze the process

The selected papers in our survey are also evaluated based on the type of the statistic/index which is used to analyze the outcome of the process. To design control charts from a statistical, economic or an economic-statistical point of view, different statistics have been used to monitor the process mean, the process variability as well as the joint process mean and variability under different assumptions and situations. The statistics in this regard are classified as follows:

- **Shewhart-type statistic:** This type of control chart which is first proposed by Shewhart in 1924 is used to monitor either variable or attribute quality characteristics. The Shewhart (or memory-less) statistics only use the information of the last sample taken from the process and they are particularly sensitive to the detection of large process shifts.
- **Memory-based statistic:** In this case the current value of the monitored statistic depends on the current observation as well as on the previous ones. This approach allows to increase the sensitivity of memory-based charts such as exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts to detect small and moderate process changes.
- **Shewhart-type & memory-based statistics:** In addition to the mentioned categories, there are few researches in which both Shewhart-type and memory-based statistics are used together to monitor different processes when the measurement errors exist.

In process capability analysis area, some attentions are also devoted to process capability analysis using different indices either in univariate or multivariate cases.

## 4. Results

To analyze the relevant researches in the literature and to illustrate the differences between them, a classification scheme under the mentioned criteria is conducted in this section.

### 4.1. Number of documents considering the year of publication

Fig. 1 illustrates the number of related papers published in the area of the measurement errors effect on SPM considering the year of publication. As shown in Fig. 1, at least three papers per year have been published from 2005 to 2016, except 2008 with only one paper. Fig. 1 shows that the number of publications has increased in recent years especially from 2001. This Figure clearly reveals an increasing interest to explore the effect of measurement errors on different areas of SPM.

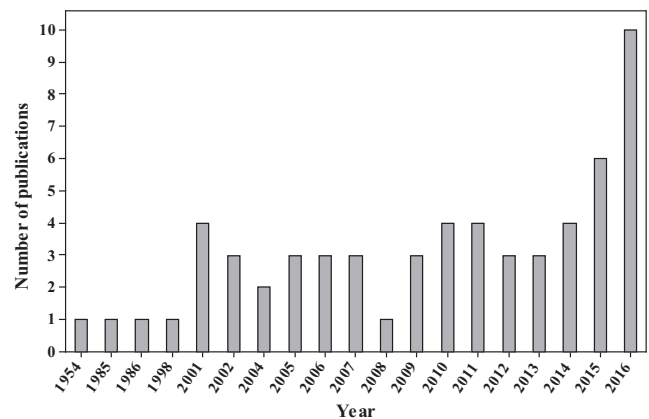


Fig. 1. Number of documents considering the year of publication.

### 4.2. The title of journal considering the number of relevant publications

The results of our analytical study concerning the second feature are summarized in Table 2. Table 2 shows that the documents are distributed over 35 different journals. The first and second journals in terms of frequency of the published papers are “Quality and Reliability Engineering International” and “Journal of Applied Statistics” with nine and six papers, respectively. These journals are followed by “Journal of Quality Technology”, “Journal of Statistical Computation and Simulation” and “The International Journal of Advanced Manufacturing Technology” with three documents for each of them.

### 4.3. The name of author/co-author considering the number of relevant publications

Table 3 lists the names of researchers who published the papers dealing with the effects of measurement errors on SPM procedures along with their affiliations and countries. As it is seen, according to the number of documents, Philippe Castagliola and Michele Scagliarini both with six papers are the most active authors dealing with this area.

### 4.4. Conceptual classification scheme to assess the relevant papers

Here, all 60 relevant papers concerning the effect of measurement errors on different areas of SPM are listed in Table 4 with respect to the presented conceptual classification scheme. The results of Table 4 are also summarized in Table 5. Note that, the reviewed papers are categorized into two time periods 1954–2006 (from when the first relevant paper is published) and 2007–2016 (i.e. the past decade). The analysis of the literature with respect to the presented conceptual classification scheme is detailed in Section 5. In Section 5, the discussions for each mentioned time periods are also provided with respect to the each question given in Table 1.

## 5. Discussion

In this section, an analytic view of the selected papers with respect to each question in Section 2 is provided.

### 5.1. Analysis of selected papers with respect to the SPM area

The distribution of the reviewed journal papers concerning the SPM area is summarized in Fig. 2. As indicated in Fig. 2, the most

**Table 2**

Top journals by considering the number of related publications.

Journal title	Number of papers	Percentage
Quality and Reliability Engineering International	9	15
Journal of Applied Statistics	6	10
Journal of Quality Technology	3	5
Journal of Statistical Computation and Simulation	3	5
The International Journal of Advanced Manufacturing Technology	3	5
IIE Transactions	2	3.33
International Journal of Production Research	2	3.33
Communications in Statistics-Theory and Methods	2	3.33
Economic Quality Control	2	3.33
IEEE Transactions on Semiconductor Manufacturing	2	3.33
International Journal for Quality Research	2	3.33
Computers & Operations Research	1	1.67
Quality Engineering	1	1.67
Quality & Quantity	1	1.67
Journal of Analytical Chemistry	1	1.67
Transactions of the Institute of Measurement and Control	1	1.67
Journal of Statistical Theory and Applications	1	1.67
Industrial Quality Control	1	1.67
Statistical Methods and Applications	1	1.67
Annals of Management Science	1	1.67
Microelectronics Reliability	1	1.67
Chinese Journal of Applied Probability	1	1.67
Journal of Manufacturing Systems	1	1.67
International Journal of Quality Engineering and Technology	1	1.67
International Journal of Metrology and Quality Engineering	1	1.67
European Journal of Industrial Engineering	1	1.67
Statistica Sinica	1	1.67
Engineering, Technology & Applied Science Research	1	1.67
International Journal of Engineering	1	1.67
Journal of Testing and Evaluation	1	1.67
ASTA Advances in Statistical Analysis	1	1.67
International Journal of Quality & Reliability Management	1	1.67
Journal of Manufacturing Systems	1	1.67
Statistical Papers	1	1.67
Asian Journal on Quality	1	1.67

**Table 3**

Top researchers by considering the number of related publications.

Author	Affiliation/country	Documents
Castagliola, P.	Université de Nantes & IRCCyN UMR CNRS 6597, Nantes, France	6
Scagliarini, M.	Department of Statistics, University of Bologna, Bologna, Italy	6
Chakraborty, A. B.	Department of Statistics, St. Anthony's College, Shillong, Meghalaya, India	4
Hu, X. L.	School of Automation, Nanjing University of Science and Technology, Nanjing, China	4
Khoo, M. B. C.	School of Mathematical Sciences, University Sains Malaysia, Malaysia	4
Khurshid, A.	Department of Mathematical and Physical Sciences, College of Arts and Sciences, University of Nizwa, Oman	4
Liao, M. Y.	Department of Finance and Banking, Yuanpei University, Hsinchu, Taiwan	4
Maleki, M. R.	Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran	4
Pearn, W. L.	Department of Industrial Engineering and Management, National Chiao Tung University, Taiwan	4
Sun, J. S.	School of Automation, Nanjing University of Science and Technology, Nanjing, China	4
Abbasi, S. A.	Department of Statistics, The University of Auckland, Auckland, New Zealand	3
Amiri, A.	Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran	3
Bordignon, S.	Department of Statistics, University of Padova, Italy	3
Ghashghaei, R.	Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran	3
Yang, S. F.	Department of Statistics, National Chengchi University, Taipei, Taiwan	3
Grau, D.	Laboratory of Applied Mathematics, Université de Pau et des Pays de l'Adour, Bayonne, France	2
Hamadani, A. Z.	Department of Industrial Engineering, Isfahan University of Technology, Iran	2
Linna, K. W.	Auburn University at Montgomery, Alabama, United States	2
Linna, K. W.	Auburn University at Montgomery, Alabama, United States	2
Linna, K. W.	Auburn University at Montgomery, Alabama, United States	2
Rahim, M. A.	Faculty of Business Administration, University of New Brunswick, Fredericton, Canada	2
Saghaei, A.	Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran	2
Shishebori, D.	Department of Industrial Engineering, Isfahan University of Technology, Iran	2
Stemann, D.	Department of Economics, University of Hagen, Hagen, Germany	2
Woodall, W. H.	Department of Statistics Virginia Tech Blacksburg, United States	2
Wu, C. W.	Department of Industrial Management, National Taiwan University of Science and Technology, Taipei, Taiwan	2

efforts i.e. 60% (36 from 60 papers) have been conducted on the statistical design of control charts in the presence of measurement errors. In the first work in this regard, [Bennett \(1954\)](#) studied the

effect of measurement errors on the  $\bar{X}$  control chart using the model  $Y = X + \varepsilon$ , where  $Y$  and  $X$  are the measured and the actual quantities, respectively, while  $\varepsilon$  is the random error term due to

**Table 4**  
Classification of SPM articles in the presence of measurement errors.

Paper	SPM area			Model	Error variance	Type of data					Process type	Remedial approach	Statistic/index
	Statistical design	E/E-S design	Process capability			Univariate	Multivariate	Attribute	Profile	Fuzzy			
Bennett (1954)	✓			Additive	Constant	✓					Ordinary	–	$\bar{X}$
Rahim (1985)		✓		Additive	Constant	✓					Ordinary	–	$\bar{X}$
Kanazuka (1986)	✓			Additive	Constant	✓					Ordinary	–	$\bar{X} - R$
Mittag and Stemann (1998)	✓			Additive	Constant	✓					Ordinary	–	$\bar{X} - S$
Stemann and Weihs (2001)	✓			Additive	Constant	✓					Ordinary	–	$\bar{X}/S, EWMA - \bar{X}/S$
Linna et al. (2001)	✓			Additive	Constant		✓				Ordinary	–	$\chi^2$
Linna and Woodall (2001)	✓			Additive	Constant/linearly increasing	✓					Ordinary	Multiple measurements	$\bar{X}, S^2$
Bordignon and Scagliarini (2001)			✓	Additive	Constant	✓					Ordinary	–	$C_p$
Yang (2002)		✓		Additive	Constant	✓					Ordinary	–	asymmetric $\bar{X} - S$
Bordignon and Scagliarini (2002)			✓	Additive	Constant	✓					Ordinary	–	$C_p$ & $C_{pk}$
Scagliarini (2002)			✓	Additive	Constant	✓					Autocorrelated	–	$C_p$
Shore (2004)	✓			Additive	Constant	✓					Ordinary	–	$\bar{X}, S^2$
Maravelakis et al. (2004)	✓			Additive	Constant/linearly increasing	✓					Ordinary	Multiple measurements	EWMA
Yang and Yang (2005)	✓			Additive	Constant	✓					Multi-stage & autocorrelated	–	Shewhart and cause-selecting charts
Pearn and Liao (2005)			✓	Additive	Constant	✓					Ordinary	Adjusting lower confidence bounds and critical values	$C_{pk}$
Pearn et al. (2005)			✓	Additive	Constant	✓					Ordinary	Adjusting lower confidence bounds and critical values	$C_p$ & $C_{pm}$
Chang and Gan (2006)	✓			Additive	Constant	✓					Ordinary	–	Shewhart
Bordignon and Scagliarini (2006)			✓	Additive	Constant	✓					Ordinary	–	$C_{pm}$
Pearn and Liao (2006)			✓	Additive	Constant	✓					Ordinary	Adjusting lower confidence bounds and critical values	$C_{PU}$ & $C_{PL}$
Yang et al. (2007)	✓			Additive	Constant	✓					Multi-stage	–	EWMA and cause-selecting charts
Huwang and Hung (2007)	✓			Additive	Constant		✓				Ordinary	–	sample generalized variance and LRT charts
Pearn and Liao (2007)			✓	Additive	Constant	✓					Ordinary	Adjusting lower confidence bounds and critical values	$C_p$
Wang (2008)			✓	Additive	Constant	✓					Ordinary	–	$S_{pk}$
Xiaohong and Zhaojun (2009)	✓			Additive	Constant	✓					Autocorrelated	–	CUSUM
Li and Huang (2009)	✓			4 types of errors	Constant		✓				Ordinary	–	Shewhart & EWMA
Shishebori and Hamadani (2009)			✓	Additive	Constant		✓				Ordinary	Adjusting lower confidence bounds and critical values	$MC_p$
Abbasi (2010)	✓			Two-component	Constant	✓					Ordinary	Multiple measurements	EWMA
Shishebori and Hamadani (2010)			✓	Additive	Constant		✓				Ordinary	–	$MC_p$
Villeta et al. (2010)			✓	Additive	Constant	✓					Ordinary	Inverse method	$C_p$ & $C_{pk}$

Scagliarini (2010)		✓	Additive	Constant	✓		Autocorrelated	–	$C_{pk}$
Costa and Castagliola (2011)	✓		Additive	Constant	✓		Autocorrelated	Multiple measurements	$\bar{X}$
Scagliarini (2011)		✓	Additive	Constant		✓	Ordinary	MANOVA-based method	multivariate $C_p, C_{pk}, C_{pm}, C_{pmk}$
Wu (2011)		✓	Additive	Constant	✓		Ordinary	–	$C_{pk}$
Grau (2011)		✓	Additive	Constant	✓		Ordinary	Adjusted lower confidence bound	$C_p''(u, v)$
Moameni et al. (2012)	✓		Additive	Constant		✓	Ordinary	–	fuzzy $\bar{X} - \bar{R}$
Maravelakis (2012)	✓		Additive	Constant/linearly increasing	✓		Ordinary	Multiple measurements	CUSUM
Wu and Liao (2012)		✓	Additive	Constant	✓		Ordinary	Generalized inference approach	$S_{pk}$
Chakraborty and Khurshid (2013a)	✓		Additive	Constant		✓	Ordinary	–	Shewhart
Chakraborty and Khurshid (2013b)	✓		Additive	Constant		✓	Ordinary	–	Shewhart
Grau (2013)		✓	Additive	Constant	✓		Ordinary	Adjusted critical value	$C_p^u(u, v), C_p^l(u, v)$
Khurshid and Chakraborty (2014)	✓		Additive	Constant		✓	Ordinary	–	Shewhart
Riaz (2014)	✓		Additive	Constant	✓		Ordinary	Adjusting the control limit coefficient	$\bar{X}, R, S^2$
Abbasi (2014)	✓		Two-component	Constant	✓		Ordinary	–	Shewhart, EWMA and CUSUM
Saghaei et al. (2014)		✓	Additive	Constant	✓		Ordinary	Multiple measurements	EWMA
Baral and Anis (2015)		✓	Additive	Constant	✓		Ordinary	–	$C_{pm}$
Chakraborty and Khurshid (2015)	✓		Additive	Constant	✓		Ordinary	–	ANOM chart
Ding and Zeng (2015)	✓		Additive	Constant	✓		Multi-stage	–	OLS and TLS based charts
Haq et al. (2015)	✓		Additive	Constant/linearly increasing	✓		Ordinary	RSS, MRSS, IRSS, IMRSS, multiple measurements	EWMA
Hu, Castagliola, Sun, and Khoo (2015)	✓		Additive	Constant/linearly increasing	✓		Ordinary	Multiple measurements	Synthetic $\bar{X}$
Noorossana and Zerehsaz (2015)	✓		Additive	Constant		✓	Ordinary	–	EWMA-3, EWMA/R, T <sup>2</sup>
Abbasi (2016)		✓	Two-component	Constant	✓		Ordinary	Multiple measurements, increasing $n$	EWMA
Amiri et al. (in press)	✓		Additive	Constant		✓	Ordinary	IQR based methods, increasing $n$	multivariate ELR
Daryabari et al. (in press)	✓		Additive	Constant	✓		Ordinary	–	MAX-EWMAMS
Ghashghaei et al. (in press)	✓		Additive	Constant	✓		Ordinary	RSS, multiple measurements	ELR
Hu, Castagliola, Sun, and Khoo (2016a)	✓		Additive	Constant/linearly increasing	✓		Ordinary	Multiple measurements	adaptive (VSS) $\bar{X}$
Hu, Castagliola, Sun, and Khoo (2016b)	✓		Additive	Constant	✓		Ordinary	Multiple measurements	adaptive (VSI) $\bar{X}$
Hu et al. (in press)		✓	Additive	Constant	✓		Ordinary	Multiple measurements	upper-sided synthetic $S^2$

(continued on next page)



Table 4 (continued)

Paper	SPM area		Model	Error variance	Type of data		Process type		Remedial approach	Statistic/index
	Statistical design	E/E-S design			Process capability	Univariate	Multivariate	Attribute		
Khatai Dizabadi et al. (in press)	↗		Additive	Linearly-increasing	↗		Ordinary	-		MAX-EWMAMS
Maleki et al. (2016)	↗		Additive	Linearly increasing		↘	Ordinary	Multiple measurements		multivariate ELR
Tran, Castagliola, and Celano (in press)	↗		Additive	Constant	↗		Ordinary	-		Shewhart-RZ

E: Economic design.

E-S: Economic-statistical design.

the measurement errors. Then, Kanazuka (1986) used the same model of Bennett (1954) and investigated the effect of measurement errors on the performance of the  $\bar{X}/R$  control charts. He also concluded that the efficiency of the  $\bar{X}/R$  control charts is strongly affected by the gauge measurement errors. Most researches in the area of statistical design of control charts have proposed to monitor either the process mean or the process variability, separately. However, recently some researchers such as Amiri et al. (in press), Daryabari, Hashemian, Keyvandarian, and Shekary (in press), Ghashghaei et al. (in press), Khatai Dizabadi, Shahrokhi, and Maleki (in press), and Maleki, Amiri, and Ghashghaei (2016) have limited their focus on the simultaneous monitoring of process mean and variability. As illustrated in Table 5, the number of reviewed journal papers in the case of statistical design of control charts under measurement errors has increased from the first period (1954–2006) with 10 papers to 26 papers in the second one (2007–2016).

Fig. 2 shows that the effect of measurement errors on economic/economic-statistical design of control charts has been investigated by 5 journal papers only (8.3% of the total reviewed papers) namely Rahim (1985), Yang (2002), Saghaei, Fatemi Ghomi, and Jaberi (2014), Abbasi (2016), and Hu, Castagliola, Sun, and Khoo (in press). It is also seen in Fig. 2 that the statistical design of control charts is followed by process capability analysis with totally 19 documents (31.7% of the total reviewed papers). The first paper which dealt with the problem of measurement errors effect on process capability analysis area is conducted by Bordignon and Scagliarini (2001). They conducted a statistical analysis on the simplest and the most common process capability index namely  $C_p$  when the observations are contaminated by measurement errors. After that, some other researches have been conducted to assess the effect of gauge measurement errors on different process capability indices in both the univariate and multivariate cases. Table 5 shows that all reviewed papers in the process capability analysis area during the period 1954–2006 are presented in the case of a univariate quality characteristic and no effort has been performed to investigate the effect of gauge measurement errors on multivariate process capability indices. However, in the second period (between 2007 and 2016), three journal papers have been published in this regard in the case of multivariate quality characteristics.

### 5.2. Analysis on the type of measurement errors model

The distribution of the reviewed papers considering the type of covariate model is illustrated in Fig. 3. As illustrated in Fig. 3, most papers concerning the effect of measurement errors on SPM (56 from 60 papers, i.e. about 93.3%) have used an additive model to define the relationship between actual and measured quantities while the effect of imprecise observations using a multiplicative model has been clearly neglected in the literature. As it can be seen in Table 5, all researches during the period 1954–2006 have been proposed based on an additive covariate model. During the period 2007–2015, four researches (9.76%) have limited their focus on the other covariate models. In this regard, Li and Huang (2009) studied the monitoring and the fault detection of multivariate processes considering four types of measurement errors, including sensor bias, sensitivity, noise and dependency of the relationship between the true and the measured values of a variable on the other ones. The effect of imprecise observations by gauge measurement errors considering TCME model is investigated by Abbasi (2010, 2014, 2016).

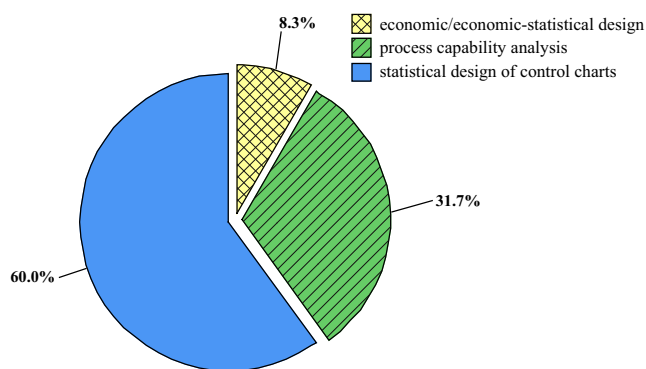
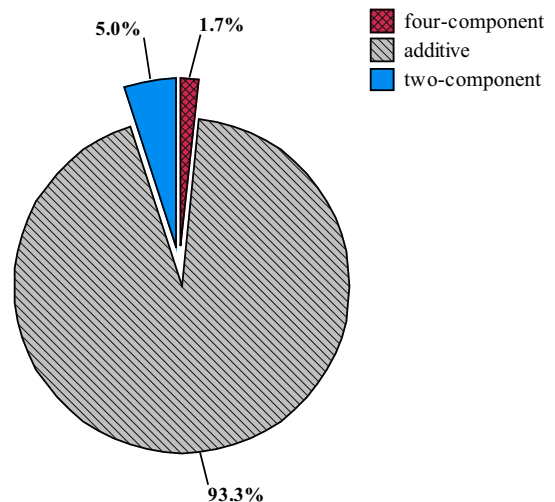
### 5.3. Analytical view on the variance of measurement errors term

The distribution of the reviewed journal papers according to the variance of measurement errors term is illustrated in Fig. 4. As seen in Table 4, all the papers in our survey from 1954 to 2011 expect

**Table 5**

Summary of the results for time periods 1954–2006 and 2007–2016.

Classification criteria	1954–2006 (19 papers)		2007–2016 (41 papers)	
	Number	Percentage	Number	Percentage
<i>What is the SPM area?</i>				
(1.1) statistical design of control charts	10	52.63	26	63.41
(1.2) economic design/economic-statistical design	2	10.53	3	7.32
(1.3) process capability analysis	7	36.84	12	29.27
<i>What type of measurement errors model is used?</i>				
(2.1) additive model	19	100	37	90.24
(2.2) multiplicative model	0	0	0	0
(2.3) four-component model	0	0	1	2.44
(2.4) two-component model	0	0	3	7.32
<i>What type of variance is assumed for the measurement error term?</i>				
(3.1) constant	17	89.47	35	85.36
(3.2) linearly increasing	0	0	2	4.88
(3.3) constant & linearly increasing	2	10.53	4	9.76
<i>Type of quality characteristic?</i>				
(4.1) univariate	18	94.74	29	70.73
(4.2) multivariate	1	5.26	7	17.07
(4.3) attribute data	0	0	3	7.32
(4.4) profile	0	0	1	2.44
(4.5) fuzzy	0	0	1	2.44
<i>Type of process?</i>				
(5.1) ordinary	17	89.48	36	87.80
(5.2) multi-stage	0	0	2	4.88
(5.3) autocorrelated	1	5.26	3	7.32
(5.4) multi-stage & autocorrelated	1	5.26	0	0
<i>What type of remedial approach is used to account for the effect of measurement errors?</i>				
(6.1) no remedial approach	14	73.68	20	48.78
(6.2) multiple measurements approach	2	10.53	9	21.95
(6.3) other approaches	3	15.79	9	21.95
(6.2) multiple measurements & other approaches	0	0	3	7.32
<i>Type of statistic/index which is used to analyze the quality of the process?</i>				
<b>(7.1) statistical, economic, economic-statistical design of control charts</b>				
7.1.1. Shewhart-type statistic	10	83.34	14	48.28
7.1.2. memory-based statistic	1	8.33	12	41.38
7.1.3. Shewhart & memory-based statistics	1	8.33	3	10.34
<b>(7.2) process capability</b>				
7.2.1. univariate index	7	100	9	75
7.2.2. multivariate index	0	0	3	25

**Fig. 2.** Distribution of the selected papers according to the SPM area.**Fig. 3.** Distribution of the selected papers according to the measurement errors model.

two ones namely Linna and Woodall (2001) as well as Maravelakis et al. (2004) assumed a constant variance for the measurement errors term. After 2011, more attentions have been paid to linearly increasing variance for the measurement errors term. Although there is an increasing trend to explore the effect of measurement errors with a linearly increasing variance, however, the constant variance case is still the most common type in both periods 1954–2006 and 2007–2016. Table 5 shows that the percentage of reviewed papers considering a linearly increasing variance for

the measurement errors term has increased from 10.53% (i.e. 2 papers) during the period 1954–2006 to 14.64% during the period 2007–2016 (i.e. 6 papers). The effect of measurement errors with linearly increasing variance on SPM is also discussed in Maravelakis (2012), Haq et al. (2015), Hu, Castagliola, Sun, and

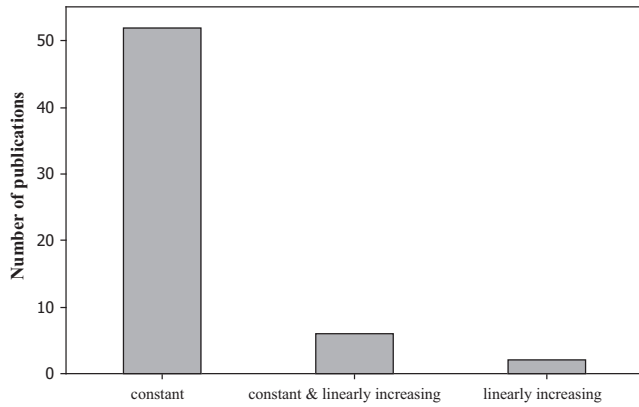


Fig. 4. Distribution of the selected papers according to variance of measurement error term.

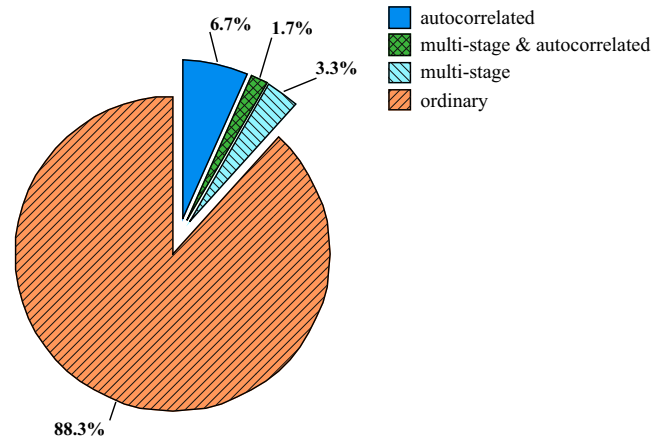


Fig. 6. Distribution of the selected papers according to the process type.

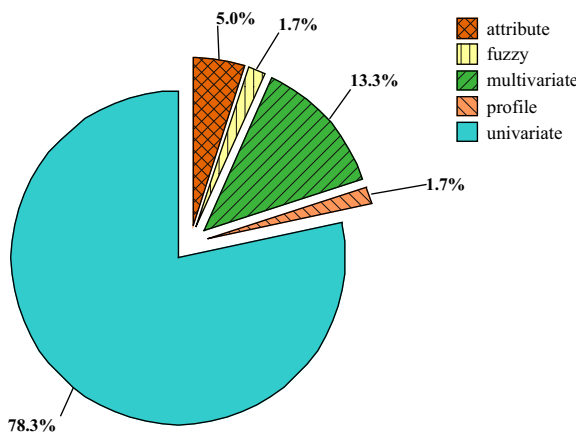


Fig. 5. Distribution of the selected papers according to the type of quality characteristic.

Khoo (2015, 2016a), Khati Dizabadi et al. (in press) and Maleki et al. (2016).

5.4. Analytical view of the literature under the type of quality characteristics

The frequency of the relevant papers according to the type of quality characteristic is shown in Fig. 5. From the first research concerning the effect of imprecise data on SPM procedures proposed by Bennett (1954), most efforts have been devoted to the univariate case, i.e. 78.3% of the reviewed journal papers. The percentage of papers which addresses multivariate quality characteristics are ranked second after the univariate case with 13.3% of the total reviewed papers. During the period 1954–2006, all the reviewed journal papers have focused on the univariate quality characteristic case, except Linna, Woodall, and Busby (2001) which is the first research concerning the effect of measurement errors on multivariate process data. Assessing the effect of imprecise observations concerning univariate quality characteristic is still the most frequent area during the period 2007–2016 with 70.73% of total reviewed papers in this period. However, more efforts in comparison with period 1954–2006 have been conducted in 2007–2016 concerning other quality situations namely multivariate, attribute, profile and fuzzy process data.

The effect of measurement errors on the performance of SPM under multivariate quality characteristics was firstly studied by

Linna et al. (2001) by considering a  $\chi^2$  control chart and assuming the following covariate model:

$$Y = A + BX + \varepsilon, \tag{6}$$

where  $X$  is a  $p \times 1$  vector of the actual quality characteristics which is assumed to follow a multivariate normal distribution with mean vector  $\mu_i$  and variance-covariance matrix  $\Sigma_p$ . In Eq. (6),  $A$  is a  $p \times 1$  vector of constants,  $B$  is an invertible  $p \times p$  matrix of constants, and  $\varepsilon$  is a  $p \times 1$  normal random vector with mean  $0$  and constant covariance matrix  $\Sigma_m$  which is assumed to be independent of  $X$ . Obviously,  $Y$ , is a  $p \times 1$  vector of measured quality characteristics which follows a multivariate normal distribution as follows:

$$Y \sim MVN(A + B\mu_i, B\Sigma_p B^T + \Sigma_m). \tag{7}$$

Linna et al. (2001) presented two bivariate examples and found that the  $\chi^2$  control chart is affected by measurement errors. The effect of gauge measurement errors on different SPM procedures considering multivariate quality characteristics is also evaluated by Huwang and Hung (2007), Shishebori and Hamadani (2009, 2010), Li and Huang (2009), Scagliarini (2011), Amiri et al. (in press), Maleki et al. (2016). Taking into account gauge measurement errors in constructing control charts to monitor attribute quality characteristics has been examined by Chakraborty and Khurshid (2013a, 2013b), and Khurshid and Chakraborty (2014).

The effect of measurement errors on monitoring fuzzy and profile quality characteristics has been addressed as follows: For fuzzy quality characteristic, Moameni, Saghaei, and Ghorbani Salanghooch (2012) investigated the effect of measurement errors on  $\tilde{X} - \tilde{R}$  fuzzy control charts to detect out-of-control situations in terms of the average run length (ARL) criterion using the linear covariate model  $Y = A + BX + \varepsilon$ . Through simulation studies, they showed that a smaller value of the measurement errors variance leads to a greater effectiveness of the  $\tilde{X} - \tilde{R}$  fuzzy control charts. They also noted that as the slope parameter of the linear covariate model increases, the effectiveness of  $\tilde{X} - \tilde{R}$  fuzzy control charts improves. Noorossana and Zerehsaz (2015) studied the effect of measurement errors on three common control charts for monitoring simple linear profiles including EWMA – 3, EWMA/R and  $T^2$  control charts in the case of random explanatory variable. They utilized a simulation study based on the ARL criterion and proved that both in-control and out-of-control performances of all considered control charts are significantly affected by the measurement errors.

### 5.5. Investigating the relevant paper according to the process type

The distribution of the reviewed papers according to the type of process is investigated in Table 4 and depicted in Fig. 6. As seen in Fig. 5, about 88.3% of the total related documents belong to the ordinary processes. However, two and four papers are devoted to the multi-stage and autocorrelated processes, respectively. As seen, only one paper is available in the literature for multi-stage process in which the observations are autocorrelated. As seen in Table 5, the percentage of the reviewed papers for ordinary and autocorrelated processes has not changed significantly from 1954 to 2006 to 2007–2016. However, the number of published documents for multi-stage processes has increased from zero during the period 1954–2006 to 2 during 2007–2016. It seems that more attentions are required by the quality engineering researchers to explore the effect of gauge measurement errors on multi-stage, autocorrelated as well as multi-stage autocorrelated processes.

The papers dealing with the impact of measurement errors on SPM methods with autocorrelated observations have been addressed by Scagliarini (2002, 2010), Xiaohong and Zhaojun (2009), and Costa and Castagliola (2011). The Effect of gauge measurement errors on monitoring multi-stage processes have been studied by Yang, Ho, and Rahim (2007) and Ding and Zeng (2015). It is also worth to mention that the effect of imprecise data caused by measurement errors on multi-stage processes in the case of autocorrelated observations has been investigated by Yang and Yang (2005).

### 5.6. Analysis of literature according to the type of remedial approach

As noted previously, it is proved in the literature that measurement errors have a severe effect in different SPM areas. Hence, to avoid misleading results, it is important to reduce such effects as much as possible. Table 5 shows that most of the reviewed journal papers during the period 1954–2006 (73.68%) have only concentrated on exploring the effects of gauge measurement errors on the performance of different SPM methods with no proposal for removing such effects. After 2006, proposing remedial approaches to decrease the effect of measurement errors on different SPM areas have attracted more attentions from quality engineering researchers such that the percentage of researches with no remedial approaches has decreased from 73.68% during the period 1954–2006 to 48.78% during the period 2007–2016.

As noted, 36 researches of the total reviewed journal papers are devoted to the effect of measurement errors on statistical design of control charts. From them, only 13 papers contain at least one remedial approach to decrease such effects, while, in 23 documents no remedial approach is available. It is worth to mention that in the area of statistical design of control charts, the researches containing at least one remedial approach has increased from 20% (i.e. 2 papers out of 10 ones) between 1954 and 2006 to 42.31% (i.e. 11 papers out of 26 ones) between 2007 and 2016. It is seen from Table 4 that the most common remedial approach to decrease the measurement errors effect on the statistical design of control charts is the multiple measurement approach, a method which is used in 11 papers from 13 ones. Multiple measurement approach to decrease the effect of measurement errors on the statistical design of different control charts has firstly been suggested by Linna and Woodall (2001). Then, Maravelakis et al. (2004) examined the effect of measurement errors on the performance of the EWMA control chart to detect out-of-control changes using the linear covariate model as in Eq. (1). They utilized multiple measurements on each sampled unit to compensate for the effect of measurement errors. For this purpose, they collected  $k$

measurements for each of  $n$  observations of  $Y$  and calculated the overall mean of these observations  $\bar{Y}$  with the following variance:

$$\frac{B^2 \sigma_X^2}{n} + \frac{\sigma_\varepsilon^2}{nk} \quad (8)$$

They also evaluated the performance of the EWMA control chart in the presence of measurement errors with linearly increasing variance. They showed that measurement errors adversely affect the performance of the EWMA control chart for monitoring the process mean. Using other remedial approaches for the statistical design of control charts in the presence of gauge measurement errors has been investigated by Riaz (2014), Haq et al. (2015), Amiri et al. (in press), and Ghashghaei et al. (in press). It is worth to mention that in Haq et al. (2015) and Ghashghaei et al. (in press), the multiple measurements method is used beside of the other remedial approaches.

Concerning the effect of imprecise data on calculating the process capability indices, it is indicated in Table 4 that 10 out of 19 relevant journal papers contain at least one remedial approach. It is interesting to find that the percentage of papers falling in the area of process capability analysis which contain at least one remedial approach has increased in recent years (from 42.86% during the period 1954–2006 to 58.33% during the period 2007–2016). The remedial approaches to compensate for the effect of measurement errors on estimating different process capability indices are discussed in Pearn and Liao (2005, 2006, 2007), Pearn, Shu, and Hsu (2005), Shishebori and Hamadani (2009), Villeta et al. (2010), Scagliarini (2011), Grau (2011, 2013), Wu and Liao (2012).

Concerning the effect of imprecise measurements on economic as well as economic-statistical design of control charts, only 2 papers are provided with no proposal to lessen such effects while, in 3 of them, namely Saghaei et al. (2014), Abbasi (2016), and Hu et al. (in press) remedial approaches to compensate for the effect of imprecise measurements are discussed.

### 5.7. Analysis of the literature according to the type of statistic/index to analyze the quality/capability of the process

The statistics/indices used to assess the quality/capability of the process are summarized in the last column of Table 4. As it can be noted in Table 5, there is an increasing trend to use memory-based statistics instead of memory-less statistics in statistical design, economic design and economic-statistical design of control charts when measurements are imprecise. The distribution of reviewed papers according to the type of statistic indicates that the percentage of memory-based statistics in statistical design, economic design and economic-statistical design of control charts has increased significantly in recent years from 16.66% (between 1954 and 2006) to 51.72% (between 2007 and 2016). In addition, all the published journal papers between 1954 and 2006 concerning the effect of measurement errors on calculating process capability indices have focused on the univariate case. However, during the period 2007–2016, 25% of the relevant documents (i.e. 3 of them) were devoted to the multivariate case. The results also confirm that  $C_p$ ,  $C_{pk}$  and  $C_{pm}$ , respectively, are the most common indices to estimate the process capability in the presence of gauge measurement errors.

## 6. Conclusions and directions for future research

In this section, research gaps related to the effect of measurement errors on different areas of SPM are highlighted. These research gaps are given as follows:

- (1) It is important to explore the effect of contaminated measurements on new SPM areas such as monitoring processes with big data, high dimensional data and social networks.
- (2) Most researches in this area have considered an additive covariate model to represent the relationship between the actual and measured quantities. More attention should be considered to other models such as the multiplicative and TCME models.
- (3) In many industrial or service systems, the quality of a process is expressed by the combination of correlated quality characteristics. However, as explained, most researches (about 92.5%) have only investigated the effect of the measurement errors on univariate processes rather than on multivariate processes. Therefore, further studies should be undertaken to investigate the effect of measurement errors on monitoring multivariate processes.
- (4) In recent years, the researchers have had considerable attempts to explore the effect of measurement errors on different monitoring schemes. However, the effect of measurement errors on change point estimators, after getting a signal from a control chart, has been neglected in the literature.
- (5) To the best of our knowledge, only two researches are available in the literature concerning the effect of measurement errors on monitoring adaptive type control charts. Consequently, this gap is a motivation for the researchers.
- (6) There is only one research in the literature for profile monitoring in the presence of measurement errors in which the simple linear profile is considered. The effect of measurement errors on the performance of control charts for monitoring the other types of profile such as multivariate profiles, generalized linear model-based profiles, multiple linear profiles, polynomial profiles can be studied in future researches.
- (7) In recent years, some efforts have been performed to explore the effect of measurement errors on simultaneous monitoring of process mean and variability. However, exploring such undesired effects by considering the other error models such as the multiplicative and TCME models is recommended.
- (8) Considering the literature, the effect of measurement errors on monitoring autocorrelated processes have been investigated in few researches. Also, these researches have considered only one quality characteristic. Consequently, exploring the effect of measurement errors on monitoring multivariate processes with autocorrelated observations is recommended. On the other hand, the effect of measurement errors on monitoring autocorrelated processes considering different time series models such as moving average (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) is suggested.
- (9) It is useful for future researches to incorporate measurement error models into self-starting control charts in which collecting sufficient large samples for Phase I analysis is not possible.
- (10) All of the papers related to process capability indices in the presence of measurement errors have assumed an additive model with constant error variance. Investigating the other measurement errors models such as the multiplicative and TCME can be a fruitful area for future researches.

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