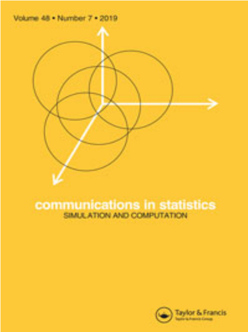


Communications in Statistics - Simulation and Computation

ISSN: 0361-0918 (Print) 1532-4141 (Online) Journal homepage: https://www.tandfonline.com/loi/lssp20

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COMMUNICATIONS IN STATISTICS - SIMULATION AND COMPUTATION®
2019, VOL. 48, NO. 7, 1964–1978
https://doi.org/10.1080/03610918.2018.1429619



# The effect of parameter estimation on phase II monitoring of poisson regression profiles

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**ABSTRACT**
The effect of parameters estimation on profile monitoring methods has only been studied by a few researchers and only the assumption of a normal response variable has been tackled. However, in some practical situation, the normality assumption is violated and the response variable follows a discrete distribution such as Poisson. In this paper, we evaluate the effect of parameters estimation on the Phase II monitoring of Poisson regression profiles by considering two control charts, namely the Hotelling's $T^2$ and the multivariate exponentially weighted moving average (MEWMA) charts. Simulation studies in terms of the average run length (*ARL*) and the standard deviation of the run length (*SDRL*) are carried out to assess the effect of estimated parameters on the performance of Phase II monitoring approaches. The results reveal that both in-control and out-of-control performances of these charts are adversely affected when the regression parameters are estimated.

**ARTICLE HISTORY**
Received 8 June 2017
Accepted 7 January 2018

**KEYWORDS**
Average run length; Parameter estimation; Poisson regression profile; Remedial methods; Standard deviation of the run length

**MATHEMATICS SUBJECT CLASSIFICATION**
62J12; 62H12

## 1. Introduction

In some statistical process monitoring applications, the quality of a product is characterized by a functional relationship between a response variable and one or more explanatory variables. Monitoring the stability of such relationships over time is referred to as profile monitoring. In the literature, control charts to monitor different types of profiles are classified into two general categories, namely Phase I and Phase II approaches. The purpose of the Phase I analysis is to estimate the unknown regression parameters using the historical data set, while in Phase II, the main interest is to quickly detect the out-of-control situations. For more information concerning profile monitoring approaches in Phases I and II, please refer to the review paper by Woodall (2007), Zhang, Li, and Wang (2009), Noorossana, Saghaei, and Amiri (2011), Xu et al. (2012), and Ghashghaei and Amiri (2017).

Previous evaluations of Phase II monitoring approaches have assumed that the in-control parameter values are known. However, in many real manufacturing on non-manufacturing environments, the process parameters are rarely known and should be estimated through an in-control data set in Phase I analysis. This issue can affect the performance of monitoring approaches due to the extra variability of the estimators especially when only a few samples are used during the Phase I analysis for estimating the process parameters. In other words, using

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estimated parameters to calculate the chart statistic when the control limits are designed for known parameters can significantly deteriorate the performance of control charts. Examples of researches concerning the effect of parameters estimation on the performance of different control charts can be found in Chakraborti and Human (2006), Chakraborti and Human (2008), Maravelakis and Castagliola (2009), Capizzi and Masarotto (2010), Zhang et al. (2011), Castagliola and Wu (2012), Zhang et al. (2013), and Rakitzis and Castagliola (2016). For more information concerning the effect of parameters estimation on the performance of different control charts, please refer to the review papers by Jensen et al. (2006) and Psarakis, Vyniou, and Castagliola (2014).

To the best of authors' knowledge, only a few researches have published papers on the effect of parameters estimation on profile monitoring approaches. As the first work in this area, Mahmoud (2012) investigated the performance of three Phase II simple linear profile approaches under estimated regression parameters in terms of the average run length (*ARL*) and the standard deviation of the run length (*SDRL*) criteria. Based on standard deviation of the average run length (*SDARL*) criterion, Aly, Mahmoud, and Woodall (2015) compared the in-control performance of three Phase II simple linear profile monitoring approaches; namely those provided by Kang and Albin (2000), Kim, Mahmoud, and Woodall (2003), and Mahmoud, Morgan, and Woodall (2010) when the regression parameters are estimated. They indicated that the method proposed by Kim, Mahmoud, and Woodall (2003) statistically outperforms the other ones in terms of the *SDARL* values. Considering the *ARL* criterion, the effect of estimated parameters on performance of EWMA-3 chart in Phase I analysis of simple linear profiles is studied by Noorossana, Aminmadani, and Saghaei (2016). Using two types of Phase I estimators, the effect of Phase I estimation on Phase II monitoring of processes with profile data was studied by Chen, Birch, and Woodall (2016).

All the above-mentioned works concerning the effect of Phase I estimators on profile monitoring approaches have assumed that the response variable follows a normal distribution and the profiles are expressed as linear models. However, in real manufacturing and non-manufacturing situations, the normality assumption of a response variable may be violated. In such situations, the relationship between the response and the explanatory variable(s) cannot be expressed by a linear relationship any longer. In this case, the generalized linear model (GLM) is used when the response variable belongs to the family of exponential distributions such as the binary, Poisson and Gamma distributions. Monitoring GLM-based profiles has been studied by some researchers: Yeh, Huwang, and Li (2009), Shang, Tsung, and Zou (2011), Amiri, Koosha, and Azhdari (2011), Amiri, Koosha, and Azhdari (2012), Paynabar and Yeh (2012), Saghaei et al. (2012), Koosha and Amiri (2013), Soleymanian, Khedmati, and Mahlooji (2013), Noorossana, Aminnayeri, and Izadbakhsh (2013), Shadman et al. (2015), Amiri et al. (2015), Panza and Vargas (2016), Qi et al. (2016), Huwang et al. (2016), Amiri, Sogandi, and Ayoubi (2016), Shadman et al. (2017), Maleki, Amiri, and Taheriyoun (2017b), and Maleki, Amiri, and Taheriyoun (2017a).

To the best of our knowledge, there is no research in the context of profile monitoring that studies how Phase I estimated parameters can affect the detection performance of control charts in Phase II, where the possible outcomes have discrete nature. On the other hand, in many statistical profile monitoring applications, the discrete response values are count data. As an example, consider the number of agglomerates which are ejected from a volcano in successive days. In this example, the count of agglomerates (the response variable) is a function of the agglomerates diameters (the explanatory variable). The motivation of this paper is to investigate the effect of estimated regression parameters on Phase II monitoring of Poisson regression profiles. Simulation studies based on the in-control average run length ($ARL_0$) and the in-control standard deviation of the run length ($SDRL_0$) are conducted to compare the in-control performances of the Hotelling's $T^2$ and MEWMA charts when estimated regression parameters are used to construct the chart statistics. For both control charts, the minimum number of Phase I sample data to ensure a predetermined value of $ARL_0$ is obtained. Also, in order to decrease the rate of false alarms when the regression parameters are estimated, the control limits of both charts are modified to reproduce the predetermined $ARL_0$. Finally, based on corrected control limits, the out-of-control performances of these charts to detect sustained shifts in regression parameters are compared for both known and estimated parameters cases.

The structure of this paper is organized as follows: the Poisson regression model along with two control charts for Phase II monitoring of Poisson regression profiles are briefly described in Section 2. The proposed scheme to evaluate the effect of estimated parameters on Phase II monitoring of Poisson regression profiles is described in Section 3. Simulation studies are applied in Section 4 to assess how using estimated parameters to construct chart statistics affects the in-control performance of both the Hotelling's $T^2$ and MEWMA charts. In Section 5, two approaches are discussed to compensate for the adverse effect of parameter estimation on the in-control performance of the extended charts. Then, in Section 6, the detection performance of the charts mentioned in the case of parameter estimation and known parameters are compared under different out-of-control scenarios considering different number of Phase I reference data. Finally, several concluding remarks and two recommendations for future studies are given in Section 7.

## 2. Phase II monitoring of poisson regression profiles

Phase I monitoring of Poisson regression profiles is studied by Amiri, Koosha, and Azhdari (2011), Amiri et al. (2015) and Maleki et al. (2017c). This section is organized in three parts. In the first subsection, the Poisson regression model is described. Afterwards, two control charts, namely the Hotelling's $T^2$ and the MEWMA charts for Phase II monitoring of Poisson regression profiles are explained in the second and third subsections, respectively.

### 2.1. Poisson regression model

Let $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ be a $p \times n$ matrix of explanatory variables where $\mathbf{x}_i = (x_{i,1}, \ldots, x_{i,p})^\intercal$, $i = 1, \ldots, n$. Let also $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^\intercal$ denote the vector of regression parameters and $\mathbf{y}_j = (y_{j,1}, \ldots, y_{j,n})^\intercal$ be the vector of response variables for profile $j = 1, 2, \ldots$ which is supposed to follow a Poisson distribution with parameter $\lambda_i$, $i = 1, \ldots, n$. Note that, it is customary in the literature that $x_{i,1} = 1$ such that $\beta_1$ is the intercept of the model. To associate the Poisson response variable to the value of the explanatory variables for the experimental setting $i$, the log link function is used for each $\lambda_i$, i.e.

$$\lambda_i = \exp(\mathbf{x}_i^\intercal \boldsymbol{\beta}), \quad i = 1, \ldots, n \tag{1}$$

### 2.2. Hotelling's $T^2$ chart

Yeh, Huwang, and Li (2009) proposed five different Hotelling's $T^2$ charts to monitor logistic regression profiles in Phase I. The Hotelling's $T^2$ chart was also used by some authors such as Amiri, Koosha, and Azhdari (2012), Koosha and Amiri (2013), and Maleki, Amiri, and Taheriyoun (2017b) to monitor different types of GLM-based profiles. The Hotelling's $T^2$

chart statistic for the Phase II monitoring of Poisson regression profiles is given as:

$$T_j^2 = (\widehat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_0)^{\mathsf{T}} \boldsymbol{\Sigma}_0^{-1} (\widehat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_0), \quad j = 1, 2, \ldots, \tag{2}$$

where $\boldsymbol{\beta}_0$ and $\boldsymbol{\Sigma}_0$ are the in-control mean vector and variance-covariance matrix of the regression parameters, respectively, and $\widehat{\boldsymbol{\beta}}_j = (\widehat{\beta}_{j,1}, \ldots, \widehat{\beta}_{j,p})^{\mathsf{T}}$ is the vector of estimated regression parameters for profile $j$. Note that the parameter estimation procedure by iterative weighted least square (IWLS) method to obtain $\widehat{\boldsymbol{\beta}}_j$ is detailed in Sharafi, Aminnayeri, and Amiri (2013). The in-control variance-covariance matrix of the regression parameters is obtained as:

$$\boldsymbol{\Sigma}_0 = (\mathbf{XWX}^{\mathsf{T}})^{-1}, \tag{3}$$

where $\mathbf{W} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$. The control chart triggers an out-of-control signal for profile $j = 1, 2, \ldots$ if $T_j^2 > h_T$, where $h_T$ is selected such that a predetermined $ARL_0$ value is achieved.

### 2.3. MEWMA chart

Zou, Tsung, and Wang (2007) proposed a Multivariate Exponentially Weighted Moving Average (MEWMA) chart for Phase II monitoring of general linear profiles. Then, Soleymanian, Khedmati, and Mahlooji (2013) used this approach to monitor GLM profiles in the case of binary response data. Here, in order to monitor the Poisson regression profile for sample point $j = 1, 2, \ldots,$ the following $p \times 1$ vector has to be defined:

$$\mathbf{z}_j = (\mathbf{XWX}^{\mathsf{T}})^{\frac{1}{2}} (\widehat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}_0). \tag{4}$$

Then, the MEWMA chart statistic is written as follows:

$$\mathbf{w}_j = \theta \mathbf{z}_j + (1 - \theta) \mathbf{w}_{j-1}, \tag{5}$$

where $\theta \in [0, 1]$ is a smoothing constant and $\mathbf{w}_0 = \mathbf{0}$. The control chart triggers an out-of-control signal for profile $j = 1, 2, \ldots$ if $\mathbf{w}_j^{\mathsf{T}} \mathbf{w}_j > h_E$, where $h_E$ is set such that a predetermined value of $ARL_0$ is obtained.

## 3. Effect of parameters estimation on monitoring poisson regression profiles

In order to investigate the effect of parameters estimation on the performance of the Hotelling's $T^2$ and the MEWMA charts to monitor Poisson regression profiles, we suggest to use the following approach:

1. Assume that the parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^{\mathsf{T}}$ are known and, based on 10000 replications, select the values of $h_T$ (for the Hotelling's $T^2$ chart) and $h_E$ (for the MEWMA chart) such that the $ARL_0 \approx 200$.
2. Generate $m$ Poisson regression profiles based on the known parameters.
3. For each sample generated in Step 2, compute the estimated vector $\widehat{\boldsymbol{\beta}}_j = (\widehat{\beta}_{j,1}, \ldots, \widehat{\beta}_{j,p})^{\mathsf{T}}$, $j = 1, \ldots, m$.
4. Compute the vector $\widehat{\overline{\boldsymbol{\beta}}} = \frac{1}{m} \sum_{j=1}^{m} \widehat{\boldsymbol{\beta}}_j$ and $\widehat{\mathbf{W}} = \text{diag}(\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_n)$ where $\hat{\lambda}_i = \exp(\mathbf{x}_i^{\mathsf{T}} \widehat{\overline{\boldsymbol{\beta}}})$, $i = 1, \ldots, n$. Then, set $RL = 1$.
5. Generate a random Poisson profile based on the known parameters.

6. Compute the value of the Hotelling's $T^2$ and MEWMA statistics based on the vector of estimated parameters $\bar{\widehat{\boldsymbol{\beta}}}$ and matrix $\widehat{\mathbf{W}}$ obtained in Step 4 and compare them with the values of the control limits $h_T$ and $h_E$ obtained in Step 1. The statistics for the Hotelling's $T^2$ and the MEWMA charts are $T_j^2 = (\widehat{\boldsymbol{\beta}}_j - \bar{\widehat{\boldsymbol{\beta}}})^\intercal \widehat{\boldsymbol{\Sigma}}_0^{-1} (\widehat{\boldsymbol{\beta}}_j - \bar{\widehat{\boldsymbol{\beta}}})$ and $\mathbf{z}_j = (\mathbf{X}\widehat{\mathbf{W}}\mathbf{X}^\intercal)^{\frac{1}{2}} (\widehat{\boldsymbol{\beta}}_j - \bar{\widehat{\boldsymbol{\beta}}})$, respectively, with $\widehat{\boldsymbol{\Sigma}}_0 = (\mathbf{X}\widehat{\mathbf{W}}\mathbf{X}^\intercal)^{-1}$ and $\widehat{\mathbf{W}} = \text{diag}(\widehat{\lambda}_1, \widehat{\lambda}_2, \ldots, \widehat{\lambda}_n)$.
7. If the values of the computed statistics in Step 6 are equal to or smaller than the corresponding control limits, then set $RL = RL + 1$ and go to Step 5; Otherwise, go to Step 8 if the value of the statistic is larger than the control limit.
8. Record the value of run length and go to Step 2.

Steps 2–8 are repeated 10000 times in order to obtain the average of the in-control run length values for several values of $m$.

## 4. Simulation studies

In this section the effect of estimated regression parameters on the performance of the Hotelling's $T^2$ and MEWMA charts for monitoring Poisson regression profiles is assessed through simulation experiments. Without loss of generality, it is assumed that when the process is in-control, $\lambda_i = \exp\{\beta_1 + \beta_2 x_i\}$, $i = 1, \ldots, n$ where $\beta_1 = 3$ and $\beta_2 = 2$. As in Amiri et al. (2015), the vector of explanatory variables, which is fixed for each sample, is considered as $\mathbf{X} = (0.1, 0.2, \ldots, 0.9)$. Also, three possible outcomes for the smoothing constant of the MEWMA chart have been selected, namely $\theta \in \{0.05, 0.1, 0.2\}$. Based on 10000 simulation runs, we have obtained $h_T = 10.8724$ (for the Hotelling's $T^2$ chart) and $h_E = 0.1945, 0.4644$ and $1.0889$ (for the MEWMA chart) when $\theta = 0.05, 0.1$ and $0.2$, respectively, in order to have $ARL_0 \approx 200$. Based on Equation (3), the variance-covariance matrix of the Poisson regression parameters is obtained as:

$$\boldsymbol{\Sigma}_0 = \begin{pmatrix} \sigma^2_{\hat{\beta}_1} & \rho\sigma_{\hat{\beta}_1}\sigma_{\hat{\beta}_2} \\ \rho\sigma_{\hat{\beta}_1}\sigma_{\hat{\beta}_2} & \sigma^2_{\hat{\beta}_2} \end{pmatrix} = \begin{pmatrix} 0.0141 & -0.0196 \\ -0.0196 & 0.0314 \end{pmatrix}$$

Table 1 presents the $ARL_0$ and $SDRL_0$ values of the Hotelling's $T^2$ and MEWMA charts for the estimated parameters case under different values of $m$. Note that the value of $m = \infty$ given in the last column of Table 1 implies that the control charts are constructed using the known regression parameters. Table 1 shows that using the estimated regression parameters to calculate the charts statistics when the control limits are set based on the known parameters can strongly affect the in-control performance of both the Hotelling's $T^2$ and MEWMA charts.

**Table 1.** $ARL_0$ and $SDRL_0$ values for different values of $m$.

| | | $m$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Chart | criterion | 5 | 7 | 10 | 15 | 20 | 30 | 40 | 50 | 70 | $\infty$ |
| Hotelling's $T^2$ | $ARL$ | 123.540 | 137.074 | 152.293 | 164.994 | 173.523 | 184.625 | 189.513 | 193.653 | 196.916 | 199.802 |
| | $SDRL$ | 148.492 | 156.115 | 169.519 | 171.816 | 181.412 | 189.172 | 193.753 | 198.796 | 205.395 | 200.340 |
| MEWMA $\theta = 0.05$ | $ARL$ | 39.187 | 46.920 | 55.595 | 70.942 | 80.776 | 93.108 | 106.485 | 115.531 | 127.259 | 201.662 |
| | $SDRL$ | 58.327 | 65.094 | 76.906 | 89.287 | 99.028 | 102.439 | 112.807 | 126.545 | 126.703 | 188.818 |
| MEWMA $\theta = 0.1$ | $ARL$ | 43.457 | 52.033 | 61.315 | 75.100 | 87.277 | 106.648 | 116.022 | 125.093 | 138.079 | 199.299 |
| | $SDRL$ | 68.749 | 75.855 | 82.837 | 90.018 | 103.326 | 119.593 | 127.633 | 134.736 | 136.480 | 194.651 |
| MEWMA $\theta = 0.2$ | $ARL$ | 49.430 | 59.438 | 74.023 | 91.547 | 100.482 | 119.639 | 132.053 | 139.350 | 154.382 | 199.067 |
| | $SDRL$ | 78.973 | 85.725 | 102.902 | 113.337 | 119.203 | 134.833 | 143.405 | 150.546 | 161.940 | 194.797 |

The results in Table 1 indicate that the effect of the estimated parameters on the MEWMA chart is stronger than on the Hotelling's $T^2$ chart, in terms of the $ARL_0$ metric. For example, when $m = 15$, we have $ARL_0 = 164.994$ for the Hotelling's $T^2$ chart, while for the MEWMA chart we have $ARL_0 = 70.942$, 75.100 and 91.547 when $\theta = 0.05$, 0.1 and 0.2, respectively. However, in the case of estimated parameters, the $SDRL_0$ values of the MEWMA chart are smaller than the one of the Hotelling's $T^2$ chart. In addition, it can be seen that for all values of parameter $m$, the MEWMA chart for $\theta = 0.2$ is the least affected one compared to the other values of parameter $\theta$. In general, we can conclude that as the value of the smoothing constant $\theta$ increases, the effect of the parameters estimation on the performance of the MEWMA chart decreases in terms of the $ARL_0$ metric. For the MEWMA chart this trend is reversed when the $SDRL_0$ metric is taken into consideration. In the other words, as the value of parameter $\theta$ increases, the dispersion of the run length values will also increase. The trend in the results of Table 1 is similar to that reported in Mahmoud (2012) which was carried out on linear profiles. In other words, similar to linear profiles, for Poisson regression profiles, the $ARL_0$ and $SDRL_0$ for $m < \infty$ are increasing when $m$ increases but they are always smaller than $ARL_0$ and $SDRL_0$ for $m = \infty$. It is worth mentioning here that, when $m$ is small, the accuracy of the estimated parameters is low. This makes the chart statistic falling outside the upper control limit more quicker. Hence, the in-control run length values would be smaller than in the case of known parameters. Consequently, selecting small values of $m$ for estimating the regression parameters leads to obtaining smaller $ARL_0$ and $SDRL_0$ values.

## 5. Remedial measures

As seen in Section 4, constructing both Hotelling's $T^2$ and MEWMA statistics based on estimated parameters adversely affects the in-control performances of these charts. To overcome the mentioned issue, two procedures are presented in the following subsections to compensate for the effect of estimated regression parameters on Phase II monitoring of Poisson regression profiles.

**Table 2.** Values of $(m, ARL_0)$ for different value of $\Delta$.

| Chart | | $\Delta$ 80% | 85% | 90% | 95% |
|---|---|---|---|---|---|
| Hotelling's $T^2$ | | (14,163.076) | (18,170.668) | (26,181.919) | (42,191.419) |
| MEWMA | $\theta = 0.05$ | (165,163.695) | (225,170.331) | (300,181.701) | (490,192.398) |
| | $\theta = 0.1$ | (140,164.786) | (180,172.182) | (220,181.569) | (405,195.790) |
| | $\theta = 0.2$ | (90,160.566) | (125,172.296) | (200,182.640) | (275,190.898) |

**Table 3.** Values of $UCL$ to achieve $ARL_0 \approx 200$.

| Chart | $m$ 5 | 7 | 10 | 15 | 20 | 30 | 40 | 50 | 70 | 90 | 140 | 165 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T^2$ | 11.9599 | 11.7224 | 11.4724 | 11.2849 | 11.2224 | 11.0724 | 10.9974 | 10.9724 | 10.9224 | 10.8849 | 10.8787 | 10.8763 | 10.8724 |
| MEWMA $\theta = 0.05$ | 0.3655 | 0.3381 | 0.3203 | 0.2945 | 0.2788 | 0.2598 | 0.2493 | 0.2414 | 0.2320 | 0.2221 | 0.2133 | 0.2086 | 0.1945 |
| MEWMA $\theta = 0.1$ | 0.7410 | 0.7015 | 0.6582 | 0.6269 | 0.6004 | 0.5707 | 0.5495 | 0.5332 | 0.5176 | 0.5043 | 0.4927 | 0.4877 | 0.4644 |
| MEWMA $\theta = 0.2$ | 1.5264 | 1.4678 | 1.4014 | 1.3264 | 1.2920 | 1.2389 | 1.2100 | 1.1889 | 1.1639 | 1.1451 | 1.1264 | 1.1170 | 1.0889 |

**Table 4.** $ARL_1$ values for different shifts from $\beta_1$ to $\beta_1 + \delta_1\sigma_{\hat{\beta}_1}$.

| | | | $\delta_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 10 | $T^2$ | | 146.402 | 38.555 | 9.223 | 3.271 | 1.712 | 1.202 | 1.046 | 1.006 |
| | MEWMA | $\theta = 0.05$ | 48.812 | 11.139 | 6.744 | 4.950 | 3.949 | 3.313 | 2.948 | 2.579 |
| | MEWMA | $\theta = 0.1$ | 46.466 | 9.326 | 5.216 | 3.793 | 3.019 | 2.504 | 2.170 | 2.016 |
| | MEWMA | $\theta = 0.2$ | 56.759 | 9.018 | 4.290 | 3.014 | 2.365 | 2.034 | 1.842 | 1.599 |
| 30 | $T^2$ | | 132.141 | 30.655 | 7.852 | 2.928 | 1.593 | 1.168 | 1.038 | 1.004 |
| | MEWMA | $\theta = 0.05$ | 24.832 | 9.420 | 5.945 | 4.420 | 3.550 | 3.034 | 2.659 | 2.251 |
| | MEWMA | $\theta = 0.1$ | 25.516 | 7.779 | 4.763 | 3.498 | 2.803 | 2.326 | 2.066 | 1.987 |
| | MEWMA | $\theta = 0.2$ | 30.674 | 7.005 | 3.906 | 2.799 | 2.244 | 1.959 | 1.727 | 1.446 |
| 50 | $T^2$ | | 126.025 | 27.202 | 7.452 | 2.853 | 1.553 | 1.150 | 1.032 | 1.005 |
| | MEWMA | $\theta = 0.05$ | 22.212 | 8.895 | 5.726 | 4.241 | 3.435 | 2.950 | 2.553 | 2.174 |
| | MEWMA | $\theta = 0.1$ | 21.712 | 7.369 | 4.593 | 3.379 | 2.718 | 2.259 | 2.038 | 1.970 |
| | MEWMA | $\theta = 0.2$ | 26.089 | 6.682 | 3.793 | 2.709 | 2.178 | 1.918 | 1.689 | 1.397 |
| 70 | $T^2$ | | 124.160 | 26.609 | 7.202 | 2.773 | 1.552 | 1.142 | 1.031 | 1.004 |
| | MEWMA | $\theta = 0.05$ | 20.902 | 8.639 | 5.553 | 4.175 | 3.385 | 2.913 | 2.481 | 2.129 |
| | MEWMA | $\theta = 0.1$ | 20.903 | 7.205 | 4.474 | 3.325 | 2.673 | 2.224 | 2.026 | 1.964 |
| | MEWMA | $\theta = 0.2$ | 24.143 | 6.550 | 3.735 | 2.682 | 2.167 | 1.907 | 1.664 | 1.371 |
| 90 | $T^2$ | | 124.192 | 26.607 | 7.238 | 2.795 | 1.554 | 1.149 | 1.029 | 1.003 |
| | MEWMA | $\theta = 0.05$ | 20.019 | 8.462 | 5.448 | 4.087 | 3.318 | 2.852 | 2.419 | 2.093 |
| | MEWMA | $\theta = 0.1$ | 19.409 | 7.088 | 4.409 | 3.287 | 2.639 | 2.212 | 2.020 | 1.947 |
| | MEWMA | $\theta = 0.2$ | 22.720 | 6.479 | 3.696 | 2.670 | 2.154 | 1.899 | 1.653 | 1.348 |
| 140 | $T^2$ | | 124.191 | 26.561 | 7.141 | 2.766 | 1.550 | 1.148 | 1.031 | 1.004 |
| | MEWMA | $\theta = 0.05$ | 19.254 | 8.245 | 5.349 | 4.013 | 3.249 | 2.792 | 2.349 | 2.073 |
| | MEWMA | $\theta = 0.1$ | 18.925 | 7.082 | 4.373 | 3.242 | 2.612 | 2.185 | 2.010 | 1.938 |
| | MEWMA | $\theta = 0.2$ | 22.705 | 6.475 | 3.662 | 2.654 | 2.143 | 1.891 | 1.635 | 1.328 |
| 165 | $T^2$ | | 124.182 | 26.346 | 7.114 | 2.773 | 1.548 | 1.147 | 1.030 | 1.003 |
| | MEWMA | $\theta = 0.05$ | 19.244 | 8.127 | 5.277 | 3.954 | 3.226 | 2.775 | 2.326 | 2.063 |
| | MEWMA | $\theta = 0.1$ | 18.922 | 7.012 | 4.344 | 3.219 | 2.598 | 2.178 | 2.009 | 1.929 |
| | MEWMA | $\theta = 0.2$ | 22.654 | 6.465 | 3.661 | 2.615 | 2.143 | 1.878 | 1.624 | 1.321 |
| $\infty$ | $T^2$ | | 124.156 | 25.975 | 7.102 | 2.762 | 1.547 | 1.141 | 1.020 | 1.003 |
| | MEWMA | $\theta = 0.05$ | 18.968 | 8.085 | 5.199 | 3.889 | 3.167 | 2.697 | 2.263 | 2.045 |
| | MEWMA | $\theta = 0.1$ | 18.835 | 6.946 | 4.298 | 3.179 | 2.558 | 2.158 | 2.004 | 1.910 |
| | MEWMA | $\theta = 0.2$ | 22.646 | 6.428 | 3.660 | 2.611 | 2.132 | 1.867 | 1.609 | 1.311 |

## 5.1. Increasing the size of the reference sample

As seen in Table 1, increasing the number $m$ of reference data in Phase I analysis reduces the effect of estimated regression parameters on the $ARL_0$ performance of both control charts. Hence, increasing the number of reference data in Phase I analysis is recommended in the literature to compensate for the effect of estimated parameters on the performance of different control charts. However, in some cases, due to economical or other restrictions, it is not possible to collect a large sample data set for estimating the process parameters. As a consequence, it is important to determine the minimum number $m$ of reference data in Phase I to ensure a desired value of $ARL_0$.

Here, through simulation studies, the minimum number $m$ of reference data in Phase I in order to have an $ARL_0$ value of at least $\Delta = 100 \times (1 - \frac{ARL_\infty - ARL_m}{ARL_\infty})$ percent for $\Delta \in \{80\%, 85\%, 90\%, 95\%\}$ are computed and listed in Table 2. As seen in Table 2, for each value of $\Delta$, the Hotelling's $T^2$ chart needs a smaller number of Phase I sample data compared to the MEWMA chart. For example, for the Hotelling's $T^2$ chart, using $m = 26$ reference data

**Table 5.** $SDRL_1$ values for different shifts from $\beta_1$ to $\beta_1 + \delta_1\sigma_{\hat{\beta}_1}$.

| | | $\delta_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 10 | $T^2$ | 177.827 | 55.226 | 11.728 | 3.243 | 1.251 | 0.520 | 0.225 | 0.082 |
| | MEWMA $\theta = 0.05$ | 167.252 | 5.412 | 1.873 | 1.017 | 0.699 | 0.519 | 0.388 | 0.496 |
| | MEWMA $\theta = 0.1$ | 121.163 | 7.208 | 1.665 | 0.939 | 0.639 | 0.535 | 0.378 | 0.154 |
| | MEWMA $\theta = 0.2$ | 138.306 | 12.314 | 1.775 | 0.917 | 0.565 | 0.358 | 0.391 | 0.490 |
| 30 | $T^2$ | 143.643 | 33.921 | 8.168 | 2.550 | 1.005 | 0.446 | 0.207 | 0.066 |
| | MEWMA $\theta = 0.05$ | 20.228 | 3.122 | 1.386 | 0.845 | 0.618 | 0.424 | 0.487 | 0.433 |
| | MEWMA $\theta = 0.1$ | 30.114 | 3.059 | 1.320 | 0.796 | 0.563 | 0.477 | 0.256 | 0.151 |
| | MEWMA $\theta = 0.2$ | 41.637 | 3.765 | 1.380 | 0.776 | 0.499 | 0.341 | 0.453 | 0.497 |
| 50 | $T^2$ | 133.505 | 28.435 | 7.418 | 2.337 | 0.935 | 0.415 | 0.180 | 0.071 |
| | MEWMA $\theta = 0.05$ | 14.156 | 2.743 | 1.304 | 0.798 | 0.577 | 0.428 | 0.503 | 0.379 |
| | MEWMA $\theta = 0.1$ | 19.682 | 2.802 | 1.262 | 0.747 | 0.575 | 0.445 | 0.217 | 0.187 |
| | MEWMA $\theta = 0.2$ | 26.380 | 3.403 | 1.296 | 0.729 | 0.455 | 0.361 | 0.466 | 0.489 |
| 70 | $T^2$ | 130.982 | 28.387 | 6.900 | 2.286 | 0.938 | 0.398 | 0.174 | 0.063 |
| | MEWMA $\theta = 0.05$ | 12.088 | 2.587 | 1.252 | 0.793 | 0.568 | 0.429 | 0.503 | 0.335 |
| | MEWMA $\theta = 0.1$ | 15.453 | 2.627 | 1.213 | 0.738 | 0.573 | 0.422 | 0.188 | 0.191 |
| | MEWMA $\theta = 0.2$ | 23.272 | 3.142 | 1.251 | 0.718 | 0.450 | 0.363 | 0.475 | 0.483 |
| 90 | $T^2$ | 132.859 | 27.244 | 6.937 | 2.270 | 0.921 | 0.418 | 0.174 | 0.061 |
| | MEWMA $\theta = 0.05$ | 10.966 | 2.565 | 1.203 | 0.784 | 0.554 | 0.453 | 0.496 | 0.291 |
| | MEWMA $\theta = 0.1$ | 13.634 | 2.588 | 1.181 | 0.723 | 0.570 | 0.413 | 0.194 | 0.232 |
| | MEWMA $\theta = 0.2$ | 20.319 | 3.048 | 1.226 | 0.719 | 0.443 | 0.372 | 0.479 | 0.476 |
| 140 | $T^2$ | 132.281 | 27.790 | 6.662 | 2.249 | 0.933 | 0.421 | 0.178 | 0.070 |
| | MEWMA $\theta = 0.05$ | 9.802 | 2.450 | 1.201 | 0.765 | 0.535 | 0.473 | 0.479 | 0.261 |
| | MEWMA $\theta = 0.1$ | 12.315 | 2.494 | 1.163 | 0.701 | 0.562 | 0.393 | 0.186 | 0.246 |
| | MEWMA $\theta = 0.2$ | 17.931 | 2.908 | 1.216 | 0.719 | 0.441 | 0.381 | 0.483 | 0.469 |
| 165 | $T^2$ | 131.243 | 26.503 | 6.580 | 2.250 | 0.902 | 0.414 | 0.178 | 0.062 |
| | MEWMA $\theta = 0.05$ | 9.248 | 2.406 | 1.195 | 0.759 | 0.528 | 0.479 | 0.469 | 0.244 |
| | MEWMA $\theta = 0.1$ | 11.908 | 2.493 | 1.141 | 0.707 | 0.564 | 0.388 | 0.185 | 0.260 |
| | MEWMA $\theta = 0.2$ | 17.532 | 2.903 | 1.178 | 0.695 | 0.449 | 0.391 | 0.486 | 0.467 |
| $\infty$ | $T^2$ | 132.637 | 27.471 | 6.850 | 2.358 | 0.950 | 0.423 | 0.191 | 0.061 |
| | MEWMA $\theta = 0.05$ | 8.946 | 2.384 | 1.161 | 0.740 | 0.528 | 0.499 | 0.441 | 0.209 |
| | MEWMA $\theta = 0.1$ | 11.155 | 2.455 | 1.133 | 0.700 | 0.556 | 0.372 | 0.176 | 0.289 |
| | MEWMA $\theta = 0.2$ | 16.850 | 2.978 | 1.199 | 0.691 | 0.458 | 0.394 | 0.488 | 0.463 |

to estimate the regression parameters can result in 90% of the desired $ARL_0$ value. This percentage can only be achieved for MEWMA chart by using $m = 300$, 220 and 200 samples when $\theta = 0.05$, $\theta = 0.1$ and $\theta = 0.2$, respectively. In addition, for the MEWMA chart, as $\theta$ increases, the number of Phase I reference data that is needed to ensure a predetermined value of $ARL_0$ decreases.

### 5.2. Modifying the control limits

As seen in Tables 1 and 2, the $ARL_0$ performances of the Hotelling's $T^2$ and the MEWMA charts for estimated parameters improve as the number $m$ of Phase I sample data increases. However, in some practical applications, it is not possible to wait too long for collecting a sufficiently large data set to achieve a desired $ARL_0$ value. Hence, in such situations, it is important to reduce the rate of false alarms without collecting a large Phase I sample data set. Here, through simulation experiments, the modified control limits of both charts are computed to have a predetermined $ARL_0$ value. The modified control limits of the Hotelling's

**Table 6.** $ARL_1$ values for different shifts from $\beta_2$ to $\beta_2 + \delta_2\sigma_{\hat{\beta}_2}$.

| | | | $\delta_2$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 10 | $T^2$ | 136.658 | 38.240 | 9.211 | 3.304 | 1.682 | 1.192 | 1.043 | 1.006 |
| | MEWMA $\theta = 0.05$ | 44.483 | 10.951 | 6.722 | 4.909 | 3.136 | 3.303 | 2.939 | 2.591 |
| | MEWMA $\theta = 0.1$ | 45.622 | 9.171 | 5.235 | 3.772 | 3.028 | 2.501 | 2.154 | 2.012 |
| | MEWMA $\theta = 0.2$ | 55.735 | 8.842 | 4.265 | 3.005 | 2.360 | 2.038 | 1.853 | 1.583 |
| 30 | $T^2$ | 120.542 | 29.173 | 7.536 | 2.869 | 1.562 | 1.152 | 1.032 | 1.005 |
| | MEWMA $\theta = 0.05$ | 24.752 | 9.349 | 5.930 | 4.397 | 3.557 | 3.033 | 2.655 | 2.234 |
| | MEWMA $\theta = 0.1$ | 25.083 | 7.679 | 4.697 | 3.503 | 2.812 | 2.326 | 2.066 | 1.990 |
| | MEWMA $\theta = 0.2$ | 29.825 | 7.049 | 3.915 | 2.788 | 2.237 | 1.953 | 1.720 | 1.436 |
| 50 | $T^2$ | 118.197 | 27.500 | 7.194 | 2.815 | 1.547 | 1.150 | 1.032 | 1.004 |
| | MEWMA $\theta = 0.05$ | 22.372 | 8.845 | 5.700 | 4.250 | 3.429 | 2.941 | 2.534 | 2.158 |
| | MEWMA $\theta = 0.1$ | 21.659 | 7.329 | 4.532 | 3.371 | 2.715 | 2.255 | 2.035 | 1.971 |
| | MEWMA $\theta = 0.2$ | 25.536 | 6.568 | 3.781 | 2.706 | 2.189 | 1.925 | 1.683 | 1.385 |
| 70 | $T^2$ | 116.110 | 26.791 | 7.151 | 2.754 | 1.551 | 1.154 | 1.026 | 1.004 |
| | MEWMA $\theta = 0.05$ | 20.996 | 8.656 | 5.554 | 4.146 | 3.368 | 2.885 | 2.478 | 2.119 |
| | MEWMA $\theta = 0.1$ | 20.158 | 7.228 | 4.482 | 3.330 | 2.678 | 2.231 | 2.031 | 1.955 |
| | MEWMA $\theta = 0.2$ | 23.864 | 6.447 | 3.730 | 2.673 | 2.157 | 1.919 | 1.665 | 1.354 |
| 90 | $T^2$ | 116.106 | 26.786 | 7.139 | 2.763 | 1.552 | 1.156 | 1.028 | 1.004 |
| | MEWMA $\theta = 0.05$ | 20.083 | 8.393 | 5.443 | 4.077 | 3.299 | 2.836 | 2.411 | 2.092 |
| | MEWMA $\theta = 0.1$ | 19.308 | 7.052 | 4.400 | 3.269 | 2.629 | 2.200 | 2.021 | 1.951 |
| | MEWMA $\theta = 0.2$ | 22.752 | 6.402 | 3.673 | 2.643 | 2.157 | 1.896 | 1.641 | 1.340 |
| 140 | $T^2$ | 116.105 | 26.784 | 7.116 | 2.762 | 1.550 | 1.150 | 1.026 | 1.004 |
| | MEWMA $\theta = 0.05$ | 19.136 | 8.182 | 5.292 | 3.983 | 3.250 | 2.797 | 2.353 | 2.069 |
| | MEWMA $\theta = 0.1$ | 18.599 | 6.971 | 4.341 | 3.245 | 2.600 | 2.189 | 2.014 | 1.936 |
| | MEWMA $\theta = 0.2$ | 22.418 | 6.343 | 3.644 | 2.640 | 2.137 | 1.882 | 1.631 | 1.321 |
| 165 | $T^2$ | 116.021 | 26.708 | 7.099 | 2.760 | 1.544 | 1.154 | 1.025 | 1.003 |
| | MEWMA $\theta = 0.05$ | 18.807 | 8.116 | 5.270 | 3.944 | 3.215 | 2.760 | 2.315 | 2.056 |
| | MEWMA $\theta = 0.1$ | 18.496 | 6.880 | 4.321 | 3.228 | 2.587 | 2.176 | 2.011 | 1.938 |
| | MEWMA $\theta = 0.2$ | 22.400 | 6.297 | 3.637 | 2.627 | 2.133 | 1.882 | 1.615 | 1.309 |
| $\infty$ | $T^2$ | 116.005 | 26.629 | 7.091 | 2.754 | 1.546 | 1.150 | 1.020 | 1.002 |
| | MEWMA $\theta = 0.05$ | 18.746 | 7.985 | 5.143 | 3.869 | 3.136 | 2.683 | 2.245 | 2.036 |
| | MEWMA $\theta = 0.1$ | 18.587 | 6.823 | 4.286 | 3.171 | 2.556 | 2.144 | 1.997 | 1.906 |
| | MEWMA $\theta = 0.2$ | 21.475 | 6.297 | 3.623 | 2.612 | 2.126 | 1.869 | 1.594 | 1.290 |

$T^2$ and MEWMA charts to achieve $ARL_0 \approx 200$ are summarized in 3. Table 3 shows that for both charts, decreasing the number of Phase I samples leads to wider control limits. For instance, for the Hotelling's $T^2$ chart, in order to achieve $ARL_0 \approx 200$ for $m = 5$, we need to widen the control limit by about 10% while, for $m = 70$ the control limit only needs to be widened by about 0.5%.

## 6. Detecting out-of-control shifts in the case of parameter estimation

In this section, using the same data set as in Sections 4 and 5, the effect of parameters estimation on the performance of the Hotelling's $T^2$ and MEWMA charts to detect different step changes in the regression parameters is investigated. Simulation experiments are carried out to compare the performances of these methods, in terms of the out-of-control $ARL$ ($ARL_1$) and out-of-control $SDRL$ ($SDRL_1$) criteria. To have a fair comparison for each value of parameter $m$, the $ARL_0$ value of the competing charts must be the same. Hence, to obtain the values

**Table 7.** $SDRL_1$ values for different shifts from $\beta_2$ to $\beta_2 + \delta_2\sigma_{\hat{\beta}_2}$.

| | | $\delta_2$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 10 | $T^2$ | 164.151 | 55.445 | 11.153 | 3.302 | 1.194 | 0.515 | 0.223 | 0.079 |
| | MEWMA $\theta = 0.05$ | 137.274 | 5.638 | 1.856 | 1.009 | 0.703 | 0.512 | 0.384 | 0.495 |
| | MEWMA $\theta = 0.1$ | 111.271 | 7.092 | 1.705 | 0.899 | 0.615 | 0.531 | 0.363 | 0.144 |
| | MEWMA $\theta = 0.2$ | 130.711 | 8.761 | 1.756 | 0.891 | 0.562 | 0.352 | 0.375 | 0.494 |
| 30 | $T^2$ | 131.846 | 33.037 | 7.697 | 2.463 | 0.976 | 0.428 | 0.181 | 0.076 |
| | MEWMA $\theta = 0.05$ | 21.287 | 3.079 | 1.378 | 0.833 | 0.610 | 0.420 | 0.488 | 0.423 |
| | MEWMA $\theta = 0.1$ | 29.291 | 3.085 | 1.288 | 0.782 | 0.579 | 0.476 | 0.254 | 0.136 |
| | MEWMA $\theta = 0.2$ | 40.438 | 3.804 | 1.352 | 0.754 | 0.496 | 0.343 | 0.454 | 0.496 |
| 50 | $T^2$ | 125.568 | 28.688 | 7.192 | 2.327 | 0.927 | 0.419 | 0.181 | 0.069 |
| | MEWMA $\theta = 0.05$ | 14.589 | 2.667 | 1.324 | 0.798 | 0.589 | 0.419 | 0.502 | 0.365 |
| | MEWMA $\theta = 0.1$ | 18.494 | 2.778 | 1.212 | 0.742 | 0.570 | 0.445 | 0.208 | 0.183 |
| | MEWMA $\theta = 0.2$ | 25.167 | 3.232 | 1.278 | 0.728 | 0.460 | 0.354 | 0.469 | 0.486 |
| 70 | $T^2$ | 121.304 | 27.716 | 6.882 | 2.211 | 0.938 | 0.427 | 0.166 | 0.064 |
| | MEWMA $\theta = 0.05$ | 11.816 | 2.587 | 1.234 | 0.778 | 0.552 | 0.436 | 0.502 | 0.324 |
| | MEWMA $\theta = 0.1$ | 14.853 | 2.658 | 1.186 | 0.731 | 0.582 | 0.426 | 0.193 | 0.211 |
| | MEWMA $\theta = 0.2$ | 23.139 | 3.099 | 1.247 | 0.720 | 0.443 | 0.361 | 0.474 | 0.478 |
| 90 | $T^2$ | 121.230 | 27.636 | 6.821 | 2.253 | 0.930 | 0.431 | 0.163 | 0.061 |
| | MEWMA $\theta = 0.05$ | 10.807 | 2.422 | 1.218 | 0.778 | 0.543 | 0.458 | 0.493 | 0.289 |
| | MEWMA $\theta = 0.1$ | 13.243 | 2.541 | 1.175 | 0.708 | 0.558 | 0.404 | 0.189 | 0.222 |
| | MEWMA $\theta = 0.2$ | 20.091 | 3.064 | 1.203 | 0.705 | 0.447 | 0.375 | 0.481 | 0.474 |
| 140 | $T^2$ | 121.111 | 27.276 | 6.642 | 2.144 | 0.919 | 0.417 | 0.165 | 0.061 |
| | MEWMA $\theta = 0.05$ | 9.542 | 2.362 | 1.187 | 0.760 | 0.534 | 0.468 | 0.479 | 0.253 |
| | MEWMA $\theta = 0.1$ | 11.793 | 2.511 | 1.175 | 0.709 | 0.557 | 0.396 | 0.175 | 0.247 |
| | MEWMA $\theta = 0.2$ | 18.023 | 3.000 | 1.220 | 0.702 | 0.432 | 0.385 | 0.484 | 0.467 |
| 165 | $T^2$ | 120.209 | 26.114 | 6.853 | 2.220 | 0.912 | 0.431 | 0.174 | 0.056 |
| | MEWMA $\theta = 0.05$ | 9.299 | 2.383 | 1.176 | 0.754 | 0.524 | 0.482 | 0.465 | 0.231 |
| | MEWMA $\theta = 0.1$ | 11.582 | 2.440 | 1.166 | 0.715 | 0.560 | 0.384 | 0.181 | 0.244 |
| | MEWMA $\theta = 0.2$ | 17.353 | 2.978 | 1.174 | 0.705 | 0.438 | 0.371 | 0.488 | 0.462 |
| $\infty$ | $T^2$ | 120.092 | 26.788 | 6.771 | 2.185 | 0.918 | 0.417 | 0.173 | 0.048 |
| | MEWMA $\theta = 0.05$ | 8.767 | 2.283 | 1.155 | 0.752 | 0.512 | 0.505 | 0.430 | 0.187 |
| | MEWMA $\theta = 0.1$ | 11.077 | 2.409 | 1.164 | 0.691 | 0.549 | 0.356 | 0.188 | 0.295 |
| | MEWMA $\theta = 0.2$ | 16.253 | 2.875 | 1.177 | 0.690 | 0.431 | 0.384 | 0.492 | 0.454 |

of $ARL_1$ and $SDRL_1$ for each value of $m$, the $UCL$ values summarized in Table 3 are used and substituted in Step 1 of the method presented in Section 3. The out-of-control performances of the Hotelling's $T^2$ and MEWMA charts under different step changes, in terms of $ARL_1$ and $SDRL_1$ are compared in Tables 4–9 for $m \in \{10, 30, 50, 70, 90, 140, 165, \infty\}$. Recall that the last four rows in Tables 4-7 imply that the $ARL_1$ and $SDRL_1$ values are obtained based on known parameters.

Table 4 contains the $ARL_1$ values under different shift magnitudes for the intercept parameter in units of $\sigma_{\hat{\beta}_1}$. As seen in Table 4, the estimated parameters to calculate the chart statistics adversely affect the detecting performance of both charts in terms of $ARL_1$. As expected, the results of the four last columns given in Table 4 show that in the case of known regression parameters, the MEWMA chart outperforms the Hotelling's $T^2$ chart for small shifts $\delta_1 \in \{0.25, 0.5, 0.75, 1\}$, while for large shifts $\delta_1 \in \{1.25, 1.5, 1.75, 2\}$, the performance of the Hotelling's $T^2$ chart is better than that of the MEWMA chart. It can be seen that, similar results are obtained when the chart statistics are calculated using estimated regression parameters for each value of $m \in \{10, 30, 50, 70, 90, 140, 165\}$. Moreover in most out-of-control scenarios, the results confirm that for the MEWMA chart, selecting $\theta = 0.2$ leads to the best performance of this chart to detect different shifts for different values of the parameter $m$. From Table 4 we can conclude that by increasing the number $m$ of Phase I reference data, smaller $ARL_1$ values are obtained. For example, for the Hotelling's $T^2$ chart for $\delta_1 = 0.5$, increasing the value of $m$ from 10 to 70, decreases the $ARL_1$ from 38.555 to 26.609. Taking into account the $SDRL_1$ metric, we can conclude from the results of Table 5 that for both control charts, increasing the number $m$ of Phase I reference data, $m$, decreases the dispersion of the run length values.

The $ARL_1$ and $SDRL_1$ values of the Hotelling's $T^2$ and MEWMA charts to detect different shifts in the slope parameter in units of $\sigma_{\hat{\beta}_2}$ are summarized in Tables 6 and 7, respectively. The simulated results imply that designing the chart statistics based on the estimated regression parameters reduces the capability of both control charts to detect sustained shifts in the slope parameter, in terms of both $ARL_1$ and $SDRL_1$ metrics. However, increasing the number of Phase I reference data improves the detection performance of both control charts, in terms of

**Table 8.** $ARL_1$ values for joint shifts from $(\beta_1, \beta_2)$ to $(\beta_1 + \delta_1\sigma_{\hat{\beta}_1}, \beta_2 + \delta_2\sigma_{\hat{\beta}_2})$.

| | | $(\delta_1, \delta_2)$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | (0.25,0.25) | (0.25,0.5) | (0.25,1) | (0.5,0.25) | (0.5,0.5) | (0.5,1) | (1,0.25) | (1,0.5) | (1,1) |
| 10 | $T^2$ | 38.834 | 9.895 | 1.726 | 10.079 | 3.476 | 1.220 | 1.696 | 1.212 | 1.009 |
| | MEWMA $\theta = 0.05$ | 11.423 | 6.843 | 3.993 | 6.897 | 5.017 | 3.367 | 3.984 | 3.360 | 2.629 |
| | MEWMA $\theta = 0.1$ | 9.612 | 5.320 | 3.049 | 5.299 | 3.845 | 2.552 | 3.056 | 2.564 | 2.028 |
| | MEWMA $\theta = 0.2$ | 8.977 | 4.435 | 2.379 | 4.450 | 3.066 | 2.057 | 2.391 | 2.061 | 1.625 |
| 30 | $T^2$ | 31.090 | 7.964 | 1.621 | 8.142 | 3.109 | 1.184 | 1.638 | 1.182 | 1.006 |
| | MEWMA $\theta = 0.05$ | 9.488 | 6.013 | 3.595 | 6.006 | 4.489 | 3.073 | 3.605 | 3.069 | 2.297 |
| | MEWMA $\theta = 0.1$ | 7.989 | 4.820 | 2.839 | 4.832 | 3.556 | 2.363 | 2.841 | 2.368 | 1.991 |
| | MEWMA $\theta = 0.2$ | 7.255 | 3.958 | 2.240 | 3.980 | 2.847 | 1.975 | 2.257 | 1.979 | 1.493 |
| 50 | $T^2$ | 28.921 | 7.869 | 1.585 | 7.818 | 2.939 | 1.164 | 1.578 | 1.181 | 1.005 |
| | MEWMA $\theta = 0.05$ | 9.115 | 5.770 | 3.446 | 5.776 | 4.320 | 2.983 | 3.475 | 2.986 | 2.207 |
| | MEWMA $\theta = 0.1$ | 7.557 | 4.632 | 2.742 | 4.599 | 3.413 | 2.279 | 2.738 | 2.284 | 1.978 |
| | MEWMA $\theta = 0.2$ | 6.774 | 3.854 | 2.212 | 3.852 | 2.766 | 1.944 | 2.207 | 1.948 | 1.425 |
| 70 | $T^2$ | 28.124 | 7.648 | 1.586 | 7.642 | 2.928 | 1.164 | 1.607 | 1.175 | 1.005 |
| | MEWMA $\theta = 0.05$ | 8.841 | 5.652 | 3.415 | 5.659 | 4.233 | 2.947 | 3.405 | 2.925 | 2.163 |
| | MEWMA $\theta = 0.1$ | 7.373 | 4.581 | 2.692 | 4.601 | 3.388 | 2.252 | 2.695 | 2.259 | 1.967 |
| | MEWMA $\theta = 0.2$ | 6.601 | 3.778 | 2.183 | 3.834 | 2.726 | 1.930 | 2.194 | 1.928 | 1.397 |
| 90 | $T^2$ | 28.132 | 7.392 | 1.578 | 7.798 | 2.898 | 1.169 | 1.607 | 1.172 | 1.005 |
| | MEWMA $\theta = 0.05$ | 8.557 | 5.513 | 3.345 | 5.533 | 4.167 | 2.883 | 3.339 | 2.879 | 2.125 |
| | MEWMA $\theta = 0.1$ | 7.216 | 4.492 | 2.655 | 4.476 | 3.326 | 2.229 | 2.657 | 2.236 | 1.960 |
| | MEWMA $\theta = 0.2$ | 6.575 | 3.768 | 2.171 | 3.779 | 2.710 | 1.917 | 2.175 | 1.922 | 1.375 |
| 140 | $T^2$ | 28.126 | 7.517 | 1.566 | 7.683 | 2.883 | 1.166 | 1.605 | 1.173 | 1.005 |
| | MEWMA $\theta = 0.05$ | 8.385 | 5.405 | 3.282 | 5.430 | 4.074 | 2.832 | 3.279 | 2.828 | 2.092 |
| | MEWMA $\theta = 0.1$ | 7.118 | 4.433 | 2.628 | 4.426 | 3.295 | 2.211 | 2.639 | 2.214 | 1.951 |
| | MEWMA $\theta = 0.2$ | 6.450 | 3.726 | 2.155 | 3.709 | 2.683 | 1.904 | 2.159 | 1.907 | 1.369 |
| 165 | $T^2$ | 28.113 | 7.478 | 1.560 | 7.617 | 2.897 | 1.170 | 1.604 | 1.171 | 1.005 |
| | MEWMA $\theta = 0.05$ | 8.281 | 5.338 | 3.248 | 5.341 | 4.027 | 2.799 | 3.259 | 2.808 | 2.089 |
| | MEWMA $\theta = 0.1$ | 7.080 | 4.385 | 2.620 | 4.399 | 3.281 | 2.210 | 2.623 | 2.204 | 1.948 |
| | MEWMA $\theta = 0.2$ | 6.442 | 3.684 | 2.159 | 3.691 | 2.661 | 1.903 | 2.154 | 1.903 | 1.359 |
| $\infty$ | $T^2$ | 28.103 | 7.643 | 1.580 | 7.584 | 2.926 | 1.159 | 1.599 | 1.171 | 1.004 |
| | MEWMA $\theta = 0.05$ | 8.135 | 5.258 | 3.179 | 5.244 | 3.931 | 2.727 | 3.189 | 2.727 | 2.048 |
| | MEWMA $\theta = 0.1$ | 7.084 | 4.381 | 2.574 | 4.379 | 3.244 | 2.183 | 2.580 | 2.179 | 1.924 |
| | MEWMA $\theta = 0.2$ | 6.442 | 3.704 | 2.148 | 3.712 | 2.686 | 1.882 | 2.146 | 1.894 | 1.339 |

**Table 9.** $SDRL_1$ values for joint shifts from $(\beta_1, \beta_2)$ to $(\beta_1 + \delta_1\sigma_{\hat{\beta}_1}, \beta_2 + \delta_2\sigma_{\hat{\beta}_2})$.

| | | $(\delta_1, \delta_2)$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $m$ | Chart | (0.25,0.25) | (0.25,0.5) | (0.25,1) | (0.5,0.25) | (0.5,0.5) | (0.5,1) | (1,0.25) | (1,0.5) | (1,1) |
| 10 | $T^2$ | 53.846 | 12.706 | 1.398 | 13.355 | 3.607 | 0.554 | 1.186 | 0.543 | 0.100 |
| | MEWMA $\theta = 0.05$ | 8.402 | 1.902 | 0.714 | 1.947 | 1.068 | 0.550 | 0.728 | 0.544 | 0.493 |
| | MEWMA $\theta = 0.1$ | 7.549 | 1.767 | 0.633 | 1.713 | 0.937 | 0.542 | 0.631 | 0.546 | 0.179 |
| | MEWMA $\theta = 0.2$ | 8.772 | 1.958 | 0.568 | 1.928 | 0.939 | 0.370 | 0.582 | 0.369 | 0.484 |
| 30 | $T^2$ | 34.972 | 8.255 | 1.078 | 8.805 | 2.710 | 0.473 | 1.009 | 0.478 | 0.079 |
| | MEWMA $\theta = 0.05$ | 3.115 | 1.406 | 0.620 | 1.413 | 0.849 | 0.430 | 0.619 | 0.444 | 0.457 |
| | MEWMA $\theta = 0.1$ | 3.374 | 1.342 | 0.581 | 1.349 | 0.818 | 0.492 | 0.592 | 0.493 | 0.146 |
| | MEWMA $\theta = 0.2$ | 4.105 | 1.433 | 0.488 | 1.418 | 0.791 | 0.345 | 0.496 | 0.347 | 0.500 |
| 50 | $T^2$ | 31.166 | 7.666 | 0.965 | 7.721 | 2.412 | 0.437 | 0.945 | 0.465 | 0.075 |
| | MEWMA $\theta = 0.05$ | 2.843 | 1.310 | 0.595 | 1.341 | 0.834 | 0.431 | 0.584 | 0.443 | 0.405 |
| | MEWMA $\theta = 0.1$ | 2.942 | 1.276 | 0.576 | 1.236 | 0.774 | 0.453 | 0.581 | 0.459 | 0.167 |
| | MEWMA $\theta = 0.2$ | 3.423 | 1.335 | 0.467 | 1.336 | 0.750 | 0.358 | 0.475 | 0.348 | 0.494 |
| 70 | $T^2$ | 28.990 | 7.527 | 0.956 | 7.401 | 2.503 | 0.450 | 1.006 | 0.452 | 0.075 |
| | MEWMA $\theta = 0.05$ | 2.687 | 1.280 | 0.571 | 1.289 | 0.806 | 0.430 | 0.574 | 0.430 | 0.369 |
| | MEWMA $\theta = 0.1$ | 2.760 | 1.246 | 0.580 | 1.261 | 0.757 | 0.440 | 0.573 | 0.442 | 0.194 |
| | MEWMA $\theta = 0.2$ | 3.314 | 1.301 | 0.457 | 1.332 | 0.734 | 0.348 | 0.466 | 0.363 | 0.489 |
| 90 | $T^2$ | 28.964 | 7.522 | 0.965 | 7.405 | 2.511 | 0.452 | 1.013 | 0.435 | 0.068 |
| | MEWMA $\theta = 0.05$ | 2.561 | 1.246 | 0.563 | 1.262 | 0.803 | 0.451 | 0.562 | 0.459 | 0.331 |
| | MEWMA $\theta = 0.1$ | 2.639 | 1.208 | 0.566 | 1.209 | 0.742 | 0.426 | 0.570 | 0.431 | 0.207 |
| | MEWMA $\theta = 0.2$ | 3.187 | 1.262 | 0.457 | 1.261 | 0.734 | 0.382 | 0.463 | 0.367 | 0.484 |
| 140 | $T^2$ | 28.931 | 7.476 | 0.948 | 7.349 | 2.451 | 0.436 | 1.017 | 0.455 | 0.074 |
| | MEWMA $\theta = 0.05$ | 2.509 | 1.217 | 0.544 | 1.229 | 0.773 | 0.460 | 0.546 | 0.469 | 0.290 |
| | MEWMA $\theta = 0.1$ | 2.593 | 1.208 | 0.572 | 1.180 | 0.736 | 0.413 | 0.574 | 0.411 | 0.224 |
| | MEWMA $\theta = 0.2$ | 3.073 | 1.241 | 0.455 | 1.262 | 0.735 | 0.371 | 0.462 | 0.378 | 0.483 |
| 165 | $T^2$ | 28.679 | 7.386 | 0.951 | 7.313 | 2.509 | 0.450 | 1.008 | 0.454 | 0.073 |
| | MEWMA $\theta = 0.05$ | 2.450 | 1.199 | 0.539 | 1.197 | 0.779 | 0.476 | 0.546 | 0.470 | 0.284 |
| | MEWMA $\theta = 0.1$ | 2.515 | 1.203 | 0.562 | 1.190 | 0.731 | 0.413 | 0.568 | 0.407 | 0.230 |
| | MEWMA $\theta = 0.2$ | 3.086 | 1.213 | 0.449 | 1.260 | 0.732 | 0.363 | 0.461 | 0.376 | 0.480 |
| $\infty$ | $T^2$ | 28.642 | 7.380 | 0.958 | 7.285 | 2.449 | 0.473 | 1.020 | 0.434 | 0.082 |
| | MEWMA $\theta = 0.05$ | 2.443 | 1.199 | 0.520 | 1.192 | 0.763 | 0.502 | 0.534 | 0.489 | 0.216 |
| | MEWMA $\theta = 0.1$ | 2.591 | 1.196 | 0.556 | 1.179 | 0.734 | 0.396 | 0.564 | 0.390 | 0.269 |
| | MEWMA $\theta = 0.2$ | 2.954 | 1.222 | 0.452 | 1.254 | 0.727 | 0.397 | 0.455 | 0.384 | 0.473 |

both $ARL_1$ and $SDRL_1$ metrics. Table 6 shows that for each value of $m$, the performance of the MEWMA chart to detect small shifts, $\delta_2 \in \{0.25, 0.5, 0.75, 1\}$ is better than the Hotelling's $T^2$ chart. However, for large shifts, $\delta_1 \in \{1.25, 1.5, 1.75, 2\}$ the Hotelling's $T^2$ chart outperforms the MEWMA chart.

Here the detecting performances of the Hotelling's $T^2$ and MEWMA charts, for different joint changes in the regression model parameters are compared in Tables 8 and 9. Similar to the previous results, the results of Tables 8 and 9 indicates that the capability of both methods in detecting joint shifts are adversely affected when the regression parameters are estimated. In the case of both known and estimated parameters, it is concluded from Table 8 that the MEWMA chart outperforms the Hotelling's $T^2$ chart for small shifts $(\delta_1, \delta_2) \in \{(0.25, 0.25), (0.25, 0.5), (0.5, 0.25), (0.5, 0.5)\}$ while, for other ones (large shifts) the performance of the Hotelling's $T^2$ chart is better than for the MEWMA chart. In addition the performance of the MEWMA chart under $\theta = 0.2$ is better than the other scenarios for the smoothing parameter. As expected, under all joint shifts, as the number $m$ of Phase I reference data increases, the value of $ARL_1$ will decrease. It can be concluded from Table 9 that for both control charts increasing the number $m$ of Phase I reference data, uniformly decreases the $SDRL_1$ values.

## 7. Conclusions and future research

In this paper, the effect of using estimated regression parameters to design the chart statistics for monitoring Poisson regression profiles has been evaluated. First of all, the effect of estimated regression parameters on the $ARL_0$ and $SDRL_0$ performances of the Hotelling's $T^2$ and MEWMA charts was assessed. The results showed that the in-control performance of both control charts are adversely affected when the chart statistics are designed based on the estimated regression parameters. At the second stage, the minimum number of Phase I reference samples to fulfill a predetermined $ARL_0$ value has been obtained for both control charts. Then, at the third stage, modified $UCL$ values for both methods were computed to have $ARL_0 \approx 200$. These modified $UCL$ values were used at the fourth stage of our work to evaluate the effect of estimated regression parameters on the out-of-control performance of the Hotelling's $T^2$ and MEWMA charts. Two metrics, namely $ARL_1$ and $SDRL_1$, were used to compare the detection performance of these methods. The results of the fourth stage of our study revealed that the out-of-control performances of both charts are seriously affected when the regression parameters are estimated. Monitoring *autocorrelated* Poisson regression profiles when the parameters are estimated based on Phase I sample data is recommended in a future work. In addition, taking into account the effect of parameter estimation to monitor logistic regression profiles can be considered as another study in the future.

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