Estimating the time of a step change in the multivariate-attribute process mean using ANN and MLE

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Abstract: In this paper, we consider correlated multivariate-attribute quality characteristics and provide two methods including a modular method based on artificial neural network (ANN) as well as maximum likelihood estimation (MLE) method to estimate the time of change in the parameters of the process mean. We evaluate the performance of the estimators in terms of some criteria in change point estimation and compare them through simulation studies. The results show that the proposed ANN-based model outperforms the MLE approach under most step shifts in the mean vector of the multivariate-attribute process.

Keywords: artificial neural network; ANN; step-change point estimation; multivariate-attribute quality characteristics; maximum likelihood estimation; MLE.

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1 Introduction

Nowadays, in competitive growing global market, it is essential that firms improve the quality of their processes or products. Therefore, they have devoted considerable time and cost to quality improvement plans. Process improvement can be successfully achieved by quick detection of disturbances and accurate identification of the assignable causes. The control charts are useful tools to distinguish between common and special causes and determine the out-of-control conditions. However, they could not determine the real time when the process has moved to an out-of-control state called as the change point. As consequence, estimating the time of change point would be useful due to the risk of misdiagnosing reduction. Note that, misdiagnosing usually leads to unnecessary adjustments of the process which is costly.

Considering the literature of change point estimation, we can conclude that most efforts have been concentrated on change point estimation of univariate processes. The earliest of which goes back to Samuel et al. (1998) who investigated the methods in step-change point estimation of univariate processes. For detailed information refer to the review paper on change point estimation methods presented by Amiri and Allahyari (2012).

There are many situations in real manufacturing systems where simultaneous monitoring of several correlated quality characteristics is necessary. Monitoring multivariate and multi-attribute processes using statistical methods and artificial neural networks (ANNs) are well-documented in the literature. However, most researches have been concentrated on monitoring multivariate processes in comparison with multi-attribute processes. Niaki and Abbasi (2005) proposed an ANN-based approach to diagnose faults in out-of-control conditions as well as to identify responsible quality characteristics for out-of-control signal. Niaki and Abbasi (2007a) developed a methodology to derive control limits of multi-attribute processes based on the bootstrap method. Niaki and Abbasi (2007b) developed a control scheme for monitoring multi-attribute processes. For this purpose, first they transformed original data such that their marginal probability distribution has almost zero skewness. Then, they estimated the covariance matrix of the transformed data and applied T^2 control chart. Niaki and Abbasi (2008a) proposed a perceptron neural network to monitor multi-attribute quality characteristics as well as diagnosing attribute(s) responsible for variation in the process mean.

Despite of many monitoring schemes available in the literature, there are few researches which have been extended for change point estimation of multivariate and multi-attribute processes in comparison with univaraite processes. In order to fill this gap, recently substantial researches have been done by several researchers on change point estimation of multivariate and multi-attribute processes. As the one of the first works, Nedumaran et al. (2000) proposed maximum likelihood estimation (MLE) approach to estimate step-change point in the multivariate normal process mean. Zamba and Hawkins (2006) proposed a step-change point estimator for multivariate processes in situations which the parameters are unknown. Zarandi and Alaeddini (2010) suggested a general fuzzy-statistical clustering approach and estimated the time of change in different types of control charts including univariate, uni-attribute and the multivariate control charts with either fixed or variable sampling strategies.

Doğu and Kocakoc (2011) estimated the time of step-change in covariance matrix of multivariate Normal processes. Doğu and Kocakoc (2013) also estimated the step-change point of multivariate normal processes when joint shifts in the mean vector and covariance matrix have been occurred. Afterwards, Hou et al. (2011) estimated change point of a multivariate normal process using |S| control chart and MLE method. Niaki and Khedmati (2014) first proposed two control charts to monitor multi-attribute processes. Then, they extended a MLE approach for the change point estimation of the parameters vector considering multivariate binomial processes. Shao and Hou (2013) proposed a two stage hybrid scheme for change point estimation in multivariate processes. Matteson and James (2014) proposed a novel change point estimation method for multivariate processes, which is based on the hierarchical clustering by using a single change point estimation procedure recursively. More recently, Kazemzadeh et al. (2015) proposed a change point estimator in multivariate linear profiles under linear drift shifts.

On the other hand, many authors applied ANNs for monitoring purposes to improve the performance of the control charts. Also, in the area of change point estimation, Chen and Wang (2004) applied an ANN to classify mean shifts from multivariate control chart signals. Li et al. (2006) used a supervised learning model for estimating the time of change as well as diagnosing the source(s) of variation in the process mean with high-dimensional data considering multiple change points. Atashgar and Noorossana (2010) proposed a neural network-based change point estimator in order to identify the change point and diagnose the variable responsible for shift in bivariate Normal distribution where the mean vector of the process was allowed to change linearly, and the covariance matrix was considered constant. Moreover, Atashgar and Noorossana (2012) without assuming a specified change type, identified the time of monotonic changes based on ANN approach with supervised learning algorithm. Ahmadzadeh and Noorossana (2008) proposed a procedure to estimate the mean change point in a multivariate process by using ANN approach. Also, Ahmadzadeh et al. (2011) developed a multivariate exponentially weighted moving average (MEWMA) control chart by using neural network to identify the step-change point and diagnose the variable responsible for the change in the multivariate process mean vector.

As mentioned former, in some production environments, both variable and attribute quality characteristics with correlation structure may jointly determine the quality of a process. As an example, consider a metal forming process with punching and bending operations. Here, the weight and thickness of metal sheet may be variable quality characteristics and the shape of the bent sheet and crumpling of the punched hole may be attribute quality characteristics. The first attribute quality characteristic may be checked with a gauge whereas the latter checked visually (Rezazadeh, 2014). To the best of our knowledge, there are few works provided in the literature in order to monitor multivariate-attribute processes, such as Doroudyan and Amiri (2013). Doroudyan and Amiri (2014) also designed a control scheme for monitoring multivariate-attribute processes. They used NORTA inverse method and transformed the data to a multivariate normal distribution. Then, they utilised multivariate control charts such as T^2 and MEWMA to monitor the transformed data. Amiri et al. (2015) proposed an ANN-based methodology to monitor variability of multivariate-attribute processes as well as to diagnose quality characteristics responsible for variation in the process. Maleki and Amiri (2015) proposed a novel monitoring scheme based on the combination of two multi-layer perceptron neural networks for simultaneous monitoring of multivariate-attribute process mean and variability. Maleki and Sahraeian (2015) utilised the discriminant analysis (DA) method to monitor the mean vector of correlated multivariate-attribute quality characteristics in the first module of their proposed approach. Then, in the second module, they introduced a method based on the combination of ANN and DA approaches to detect mean shifts and diagnose the source(s) of signal.

Despite of few researches for monitoring multivariate-attribute processes, there is only one research available in the literature for estimating the time of change in such processes. In the only research in this case, Maleki et al. (2015) proposed an ANN-based model to estimate change point in the multivariate-attribute process variability. They also extended MLE approach and then provided a comparison study between ANN and MLE estimators. However, estimating the time of step change in the multivariate-attribute process mean is neglected by researchers. In order to fill the gap, we propose two approaches to estimate the step-change point in the mean vector of multivariate-attribute processes in phase 2. In the first proposed method, a modular neural network-based methodology is suggested in order to estimate the time of change in the mean vector of multivariate-attribute processes. The contributions of our proposed modular methodology are listed as follows:

In the proposed ANN-based method, we use the output values of the neural network used for detection purpose as the input values of the modular neural networks that are designed for change point estimation purpose. In the other words, the ANN in modules 1 and 2 are linked in our research. The proposed method considers correlated variable and attribute quality characteristics while the other researches in the area of change point estimation only investigated Normal quality characteristics. The proposed method can be applied generally in processes with different number of quality characteristics. The architecture of the ANN modules is different from those designed in the literature of change point estimation using ANN. We consider the output with maximum value as the time that the first out-of-control sample is manifested. In the second approach, an MLE method is applied for change point estimation purpose. The performance of the proposed methods is also compared in terms of accuracy and precision criteria.

The organisation of the rest of the paper is as follows: Section 2 discusses the proposed modular ANN-based methodology to estimate the step-change point. In Section 3, the procedure of the proposed ANN-based methodology for estimating change point in the mean vector of the process is illustrated. Then in Section 4, the MLE approach is extended. In Section 5, the performance of two approaches using Monte

Carlo simulation is evaluated and compared. Section 6 presents an illustrative example for proposed approaches. Finally, the conclusion remarks as well as the future research recommendations are provided in Section 7.

2 Proposed ANN model of change point estimation

In this section the proposed ANN-based framework including two modules with the aim of estimating the time of change in the multivariate-attribute process mean is introduced. In the first module, we apply an ANN in the literature provided for detecting mean shifts as well as diagnosing quality characteristics that cause out-of-control signals. Then, in the second module, based on the results of module 1 in fault diagnosis, we propose an ANN-based model for change point estimation of mean shifts in multivariate-attribute processes where the quality characteristics are correlated.

2.1 Module 1: fault detection and fault diagnosis in the process mean

In module 1 of this research, we apply a three-layer perceptron neural network proposed by Maleki et al. (2012) for monitoring the mean vector of multivariate-attribute processes. The number of output nodes in their proposed ANN was equal to the total number of quality characteristics plus one. For example in a multivariate-attribute process where p variables and q attributes exist, they considered totally $p + q + 1$ nodes in the output layer of the proposed ANN. The first output is used to determine the process mean state (fault detection) while the others are used to diagnose quality characteristic(s) contributing for the out-of-control signals. Based on simulation experiments, they set threshold values for the output nodes of the proposed ANN. If the observed value of the first output node is greater than its threshold, an out-of-control signal is triggered by the proposed ANN. In such situations, if the observed value of each output node is greater than its corresponding threshold value, the corresponding quality characteristic is identified as the source of variation in the process mean. Hence, after detecting an out-of-control signal, totally $\binom{p+q}{j}$, $j = 1, 2, \ldots, p + q$ out-of-control situations can be diagnosed in which j quality characteristics contribute to the out-of-control signals. Consequently, there are totally $2^{p+q} - 1$ out-of-control situations in the process mean.

2.2 Module 2: change point estimation

In this section, the proposed model that is based on a set of ANNs for estimating the real time of step-change in the mean vector of multivariate-attribute quality characteristics is described. The proposed modular model consists of some ANNs equal to the total number of out-of-control situations. Diagnosing quality characteristics which contributed to out-of-control signals in module 1 is equivalent to determining which out-of-control state has been occurred. Obviously, in a multivariate-attribute process whose quality is characterised by the combination of p variables and q attributes, we design totally $\sum_{i=1}^{p+q}\binom{p+q}{i} = 2^{p+q} - 1$ ANNs for estimating different possible change points. Then, based on the diagnosed out-of-control state in module 1, one of the designed ANNs in module 2 will be activated for change point estimation in the mean vector of multivariate-attribute quality characteristics. One of the most issues that affect the performance of an ANN is choosing proper architecture. Due to the successful applications of multi-layer perceptron neural networks in various scopes of statistical process control, this architecture of networks is used in designing the set of ANNs in the proposed change point estimation model. In order to determine the number of nodes in the input layer of each ANN estimator in the proposed modular model, the following steps should be accomplished:

Suppose that k^{th}; $k = 1, 2, \ldots, 2^{p+q} - 1$ out-of-control situation has been occurred in which u; $u = 1, 2, \ldots, p + q$ quality characteristics have been diagnosed as the sources of variation by module 1. We denoted the set of out-of-control and in-control quality characteristics respectively as follows:

$$\mathbf{u} = (qc_1', \ldots, qc_u') \textit{ and } \mathbf{v} = (qc_1, \ldots, qc_v); u + v = p + q \quad (1)$$

In such out-of-control situations, first we generate random samples of size n from a multivariate-attribute process where a predetermined small shift in the mean value of out-of-control quality characteristics has been occurred. Then, we enter the generated samples into the ANN of module 1 until an out-of-control signal is received. This process is repeated 10,000 times and in each replicate we record the run length (RL) values obtained from module 1 and save them in a vector like $\mathbf{c_k}$. Finally, we select an element in vector $\mathbf{c_k}$ namely m_k such that the value of $pr(RL > m_k)$ obtained from 10,000 simulation replicates be roughly equal to zero. Then we put m_k nodes in the input layer of the neural network required for change point estimation corresponding to the k^{th} out-of-control situation. Note that, the input vector corresponding to a given sample that is entered to the neural network in module 1 is a column vector as follows:

$$\mathbf{S} = [s_1, s_2, \ldots, s_{p+q}]^T \quad (2)$$

For each $2^{p+q} - 1$ ANNs of module 2, the number of output nodes is considered as equal as to the number of input nodes. It is pointed out in the literature that there is no standard guideline for determining the number of hidden layers as well as the number of nodes in each hidden layer (Maleki et al., 2015). It is also stipulated that one or two hidden layers will be sufficient in designing most ANNs. We finalise the number of hidden layers as well the number of nodes in each one after trial and error experiments. We also use the sigmoid function as the transfer function of all ANNs which makes the outputs in the range of [0, 1].

2.2.1 Training procedure of ANNs

The most substantial issue in training process of each $2^{p+q} - 1$ ANN modules is to prepare proper training dataset. The training procedure of k^{th}; $k = 1, 2, \ldots, 2^{p+q} - 1$ ANN corresponding to the k^{th} out-of-control situation which contains m_k nodes in its input and output layers is presented as follows: Note that, in the proposed change point estimation model, the first output value observed in ANN of module 1 is used as the input values of the ANN estimators in module 2.

Consider the run length value obtained from ANN in module 1 is equal to h; $h = 1, 2, \ldots, m_k$. First of all, we generate $m_k - h$ in-control random samples of size n. Then, we simulate h random samples of size n from a multivariate-attribute process where k^{th}

out-of-control situation has been occurred. In the next step, for each of m_k random samples, we calculate vector S (the input vector of ANN in module 1) according to equation (2). Now, we enter the generated m_k vectors into the ANN of module 1 and record the observed values of the first output node. Then, we arrange these m_k values in a column vector such that the first $m_k - h$ elements are related to the in-control situation while the last h elements are related to the k^{th} out-of-control situation. Then, we consider the generated column vector as the input vector of k^{th} ANN module as follows:

$$o_k = \left(o_1, \ldots, o_{m_k - h}, o_{m_k - h + 1}, \ldots, o_{m_k}\right)^T . \tag{3}$$

We pursue this process for each value of RL, $RL = 1, \ldots, m_k$ 100 times. Obviously, for each value of possible RL in the range of 1, 2, …, m_k, we will have 100 column vectors where each vector contains m_k elements. Finally, $100 \times m_k$ column vectors each of size $m_k \times 1$ will be available as the input datasets required for training k^{th} ANN module. Note that, in order to generate out-of-control data in training stage, the step shifts in which the mean value of the quality characteristics has been increased 2σ is used.

The target vector of k^{th} ANN; $k = 1, 2, \ldots, 2^{p+q} - 1$, is a $m_k \times 1$ vector whose elements are zero except one element that is equal to one. The location of element one in the target value represents the time when in-control state is terminated, i.e., the first out-of-control sample is placed. After generating the input vectors as well as their corresponding target vectors, using back-propagation algorithm which is a supervised training algorithm, the ANNs are trained. In this research, Gaussian copula method proposed by Cherubini et al. (2004) is applied for generating training as well as the test dataset. The following pseudo code summarises the required steps for designing the proposed ANN change point estimators:

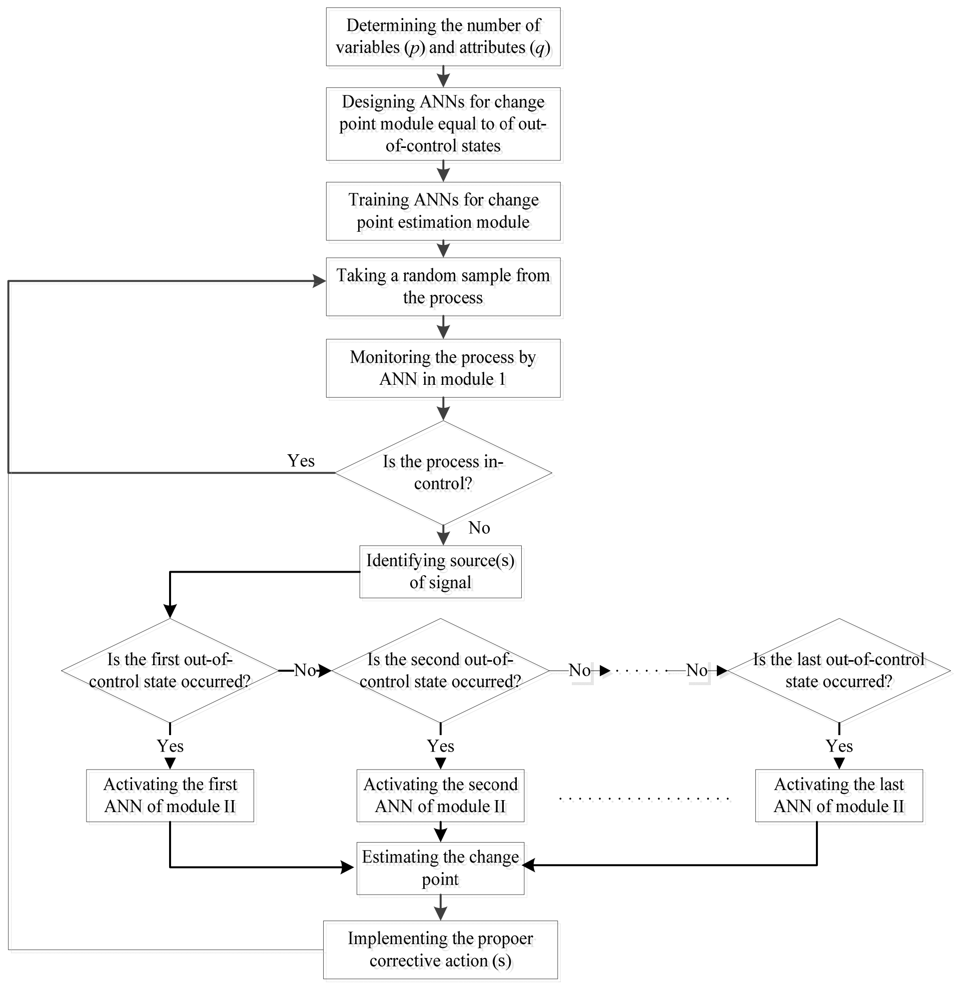
Figure 1 The pseudo code of designing each ANN change point estimator

1. Diagnose the quality characteristic(s) responsible for out-of-control signal by using ANN in module 1
2. Select the proper architecture for the ANN change point estimator
3. Determine the proper number of input nodes in the ANN change point estimator based on the results of run length obtained from ANN in module 1 for monitoring purpose.
4. Set the number of output nodes as equal as to the number of input nodes
5. Decide on the number of hidden layers and the corresponding number of nodes
6. Simulate proper input training datasets
7. Set the target vectors of input datasets
8. Train the ANN change point estimator by using back-propagation algorithm

3 The procedure of using the proposed modular ANN method

After designing and training all ANN estimators, based on the diagnosed quality characteristic(s) in module 1, we activate one of $2^{p+q} - 1$ ANNs for estimating the time of actual change in the multivariate-attribute process mean. In module 1, after detecting an out-of-control situation, we diagnose the quality characteristic(s) responsible for variation. As noted, diagnosing out-of-control factor(s) by module 1 is equivalent to determining which out-of-control control situation in the process mean has been occurred. Then, based on the diagnosed quality characteristic(s), only one ANN is activated for change point estimation in the quality characteristic(s) whose mean is (are) shifted. The proposed modular change point estimation methodology is depicted in Figure 2. As noted the target values of ANN estimators are zero and one. However, when each ANN is applied, the observed output values are not exactly equal to zero or one and they are located in the range of [0, 1] due to errors. Hence, in order to overcome this problem, the maximum observed value of the ANNs is considered as the time that the process starts to going to an out-of-control situation. For example in an ANN with 50 nodes in its output layer, if the maximum observed value is located in 35th element of output vector, the estimated time of change by the neural network is obtained equal to $\hat{\tau} = 34$.

Figure 2 The proposed modular model for change point estimation



4 MLE change point estimator

In this section, the MLE approach is used to estimate change point in the multivariate-attribute process mean. In this regard, it is assumed that the samples are coming from a process where the quality of the process or the product are expressed by both attribute and variable quality characteristics which are correlated. In addition, we assume that the mean vector and covariance matrix of in-control data are estimated using historical dataset in phase 1. It should be noted that, using normal to anything (NORTA) inverse method the data of the process are transformed in order to obtain multivariate Normal distribution [see Niaki and Abbasi (2008b) for details]. After transforming the data, let $\mathbf{x}_{ij} = (x_{ij1}, x_{ij2}, \ldots, x_{ij(p+q)})^T$ is the $p+q$-dimensional Normal distribution vector which represents the $p + q$ transformed quality characteristics on the j^{th}; $j = 1, \ldots, n$ observation of i^{th} subgroup. Also, n denotes the subgroup size and $\overline{\mathbf{x}}_i$ denotes the mean vector of the i^{th} subgroup according to the following equation:

$$\overline{\mathbf{x}}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}. \tag{4}$$

Assume further that when the process is in-control, the $\mathbf{x}_{ij}$'s are independent and identically distributed and follow a $p+q$-variate Normal distribution with mean vector of $\boldsymbol{\mu}_0$ and covariance matrix of $\boldsymbol{\Sigma}_0$. For i^{th} sample, the T^2 statistic which is defined by equation (5) is computed and plotted on the control chart. The upper control limit (*UCL*) for the extended T^2 control chart is equal to $\chi^2_{p+q,\alpha}$ which is the α percentile of the chi-square distribution with $p + q$ degrees of freedom. The process would be out-of-control when the T^2 statistic exceeds the *UCL*.

$$T_i^2 = n\left(\overline{\mathbf{x}}_i - \boldsymbol{\mu}_0\right)^{\mathrm{T}} \sum\nolimits_0^{-1}\left(\overline{\mathbf{x}}_i - \boldsymbol{\mu}_0\right). \tag{5}$$

Since we consider a step shift in the mean vector, the parameters of mean vector remain at the new level until the source of the assignable cause is identified and omitted. Hence, the parameters of $\boldsymbol{\mu}$ are equal to the corresponding in-control state, $\boldsymbol{\mu}_0$ for $i = 1, 2, \ldots, \tau$ and they become equal to unknown parameters of $\boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$ for subgroups $i = \tau + 1, \tau + 2, \ldots, T$ after elapsing unknown time. T is the last subgroup sampled. There are two unknown values τ and $\boldsymbol{\mu}_1$ which represent the last subgroup taken from an in-control process and the out-of-control process mean vector, respectively in the likelihood function. To estimate these unknown parameters along with the change point, we use the MLE approach. MLE of τ (namely $\hat{\tau}$) is the value of this parameter such that maximises the likelihood function of observations [see Nedumaran et al. (2000) for more information]. The proposed change point estimator using MLE approach can be calculated as follows:

$$\hat{\tau} = \arg\max\{M\}, \quad t = 0, 1, \ldots, T-1, \tag{6}$$

The value of M_t in equation (6) can be found as follows:

$$M_t = (T-t)\left(\overline{\overline{\mathbf{x}}}_{\mathbf{t},T} - \boldsymbol{\mu}_0\right)^T \sum\nolimits_0^{-1}\left(\overline{\overline{\mathbf{x}}}_{\mathbf{t},T} - \boldsymbol{\mu}_0\right), \tag{7}$$

where

$$\bar{\bar{\mathbf{x}}}_{\mathbf{t},T} = \frac{\sum_{i=t+1}^{T} \bar{\mathbf{x}}_i}{T-t}. \tag{8}$$

5 Performance evaluation

In this section, we present a numerical example in order to evaluate the performance of the proposed modular ANNs as well as the extended MLE approach. We consider a variable-attribute quality characteristics defined with Poisson and Normal distributions with known parameters ($p = q = 1$). In all simulations, the sample sizes of $n = 10$ are used for simulation studies under both methods. In the extended MLE approach, first using NORTA method, 100 subgroups are randomly generated from in-control Poisson and Normal distributions with parameters $\lambda_0 = 4$ and (μ_0, σ_0) = 3, 2 where the correlation coefficient between the quality characteristics is considered equal to 0.357. We apply NORTA inverse method to obtain bivariate Normal characteristics. By simulation experiments in 10,000 replicates, we set $UCL = 10.59$ in the extended T^2 control chart such that the obtained ARL_0 become roughly equal to 200. Through the simulation experiments, we estimate the mean vector and covariance matrix of the transformed data $\boldsymbol{\mu}_0$ and Σ_0 as follows:

$$\boldsymbol{\mu}_0 = (0.256, 0.0005)^T \ \boldsymbol{\Sigma}_0 = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} 0.9288 & 0.3399 \\ 0.3399 & 1.0002 \end{pmatrix}.$$

Moreover we obtain T_i^2 statistic using estimated ($\boldsymbol{\mu}_0$, Σ_0) through 10,000 replicates for $i = 1, 2, ..., 100$. In these subgroups, the statistic(s) corresponding to the in-control subgroups which exceed the UCL are discarded and replaced with the in-control statistic(s). This process repeats until all 100 generated in-control subgroups have statistics lower than the UCL. Starting with subgroup 101, observations are randomly generated under a given mean shift. Then, the out-of-control samples are plotted on the extended T^2 control chart until an out-of-control signal is received. After that, using equation (6) the change point estimate is obtained.

Now, we design a modular system consisted of three ANNs. The first ANN (called network A) is used for estimating the change point when the mean of x_1 is changed. The second ANN (called network B) is used to estimate the time of the change when the mean shift in x_2 is occurred. The third ANN (called network C) is applied for estimating change point in which simultaneous mean shifts in both x_1 and x_2 are occurred. Using ANN in module 1, the value of $\Pr(RL > 40)$ under a small mean shift in quality characteristic x_1 with the magnitude of ($\mu_1 + 0.5\sigma_1$) under 10,000 simulation replicates is obtained equal to 0.003. Hence, it seems reasonable that putting 40 nodes in the input layer of the neural network A is a proper choice. Note that increasing the number of nodes in ANNs make them too complicated and leads an exponentially increasing in the training time. By trial and error experiments, we consider only one hidden layer with 20 nodes in the neural network A. The number of nodes in the output layer of neural network A is also considered equal to 40. The designing process of the other ANNs (networks B and C), is almost similar to the network A. Using ANN of module 1, the probability $\Pr(RL > 40)$ under the shift with magnitude of ($\mu_2 + 0.5\sigma_2$) under 10,000 simulation replicates is obtained equal to 0.012. Hence, we consider 40 nodes in the input and output layers of the neural network B. Similarly, the probability value of Pr(RL > 36) under the joint shift with the magnitude of (μ_1 + 0.5σ_1, μ_2 + 0.5σ_2) under 10,000 simulation replicates is obtained equal to 0.004. Consequently, it seems that fixing 36 nodes in the input and output layer of network C is reasonable. By trial and error experiments, we consider a single hidden layer in both networks B and C with 20 and 18 nodes, respectively.

According to section 2, the required input vectors of the neural networks A and B as well as their corresponding target vectors are both the column vectors with 40 elements. In training process of neural network A, in order to generate out-of-control data we use the increasing shifts with the magnitude of 2σ_1 in the mean of x_1. Similarly, in order to generate out-of-control data required for training network B the increasing shift with the magnitude of 2σ_2 in the mean of x_2 is used. The input and target vectors that are prepared for training the network C are the column vectors with 36 elements. We also use joint step shifts in the mean value of both x_1 and x_2 with the magnitude of (2σ_1, 2σ_2). Hence, 4,000 input data for training networks A and B and 3600 input data required for training the network C are available. Finally, we train all the ANNs using back-propagation training algorithm. The value of mean square error (MSE) which is an evaluation criterion in training stage for the ANNs A, B and C are obtained equal to 0.0125, 0.0124, and 0.0134, respectively. When each trained ANN is applied for estimating the time of change in the corresponding quality characteristics(s), we focus on the maximum observed output and consider it as the estimation of time when the first out-of-control sample is manifested in the process.

The results of applying both methods in estimation of change point in the mean vector of the process under different step shifts based on 10,000 simulation studies are reported in Tables 1, 2 and 3. The MATLAB codes leading to the results of the paper are available for readers upon the request. Tables 1 and 2 are organised for separate mean shifts in x_1 and x_2 quality characteristics, respectively while the last shows the results of simultaneous shifts in both quality characteristics. We compare the performance of both methods in terms of the mean as well as the standard deviation of difference between the change point estimate and exact change point ($|\hat{\tau} - \tau|$) under different magnitudes of step mean shifts $\boldsymbol{\delta}$ = ($\delta_1\sigma_1$, $\delta_2\sigma_2$). Tables 1–3 also provide the precisions of both change point estimation methods in terms of the probability which lies in the specified tolerance. This criterion is demonstrated with $p(|\hat{\tau} - \tau| \le i)$ where i is considered equal to 0, 1, 2, 3, 4 and 5. For example, the mean and standard deviation of difference between the exact change point and the estimate obtained by the ANN-based estimator under the shift (σ_1, 0) are 0.253 and 0.672, respectively. Table 1 demonstrates that in almost all situations, the mean and standard deviation of difference between the exact change point and the estimate obtained by both methods decrease as the magnitude of the step mean shifts increases.

In addition, in 94.50% of simulation runs under the shift with the magnitude of (σ_1, 0), the difference between the estimates obtained by the neural network A and the exact change point is equal or less than 1. In other words, the probability that $\hat{\tau}$ lies within tolerance of 1 or less from the real change point is 0.945 that named as precision 1. Using the extended MLE approach this value is obtained equal to 94.26 %. Moreover, using ANN approach in 83.20% of the simulation runs, the estimator correctly identifies the real time of the change, while this value is obtained equal to 81.70 % under the extended MLE approach. In general from Table 1, we can conclude that the MLE approach performs more precise under the shifts ($0.5\sigma_1$, 0), ($0.75\sigma_1$, 0), while in the other shifts the ANN approach outperforms the MLE approach. Note that the performance of both methods under the large shift of ($2.5\sigma_1$, 0) is almost similar. Also, we notice that using MLE approach the percentage of the simulation trials identifying the change point correctly are 46.29%, 68.07%, 81.70%, 88.98%, 93.76%, 96.58%, 98.17% and 99.7% for shift magnitudes of ($0.5\sigma_1$, 0), ($0.75\sigma_1$, 0), (σ_1, 0), ($1.25\sigma_1$, 0), ($1.5\sigma_1$, 0), ($1.75\sigma_1$, 0), ($2\sigma_1$, 0) and ($2.5\sigma_1$, 0) respectively. The percentage of exact estimation for such shifts using the proposed modular ANN-based method are obtained equal to 16.65%, 51.85%, 83.20%, 95.00%, 97.60%, 98.30%, 98.40% and 98.15%, respectively. Clearly, from both methods the probability of exact estimation increases as the magnitude of the step shift increases.

On the whole, as the magnitude of the step change increases, the performance of both methods in terms of $p(|\hat{\tau}-\tau| \le i)$, i = 0, 1, …, 5 improves significantly. Analysing the results of Tables 2 and 3 is also similar to Table 1.

Table 1 Comparison between neural network A and MLE approach under different shifts in the mean of first quality characteristic

Shift	*($0.5\sigma_1$, 0)*		*($0.75\sigma_1$, 0)*		*(σ_1, 0)*		*($1.25\sigma_1$, 0)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\lvert\hat{\tau}-\tau\rvert)$	5.675	1.609	1.25	0.815	0.253	0.649	0.062	0.419
$Std(\lvert\hat{\tau}-\tau\rvert)$	6.460	0.043	2.17	0.038	0.672	0.046	0.344	0.037
$p(\lvert\hat{\tau}-\tau\rvert = 0)$	16.65	46.29	51.85	68.07	83.20	81.70	95.00	88.98
$p(\lvert\hat{\tau}-\tau\rvert \le 1)$	30.65	70.60	72.35	88.15	94.50	94.26	99.70	96.36
$p(\lvert\hat{\tau}-\tau\rvert \le 2)$	42.25	82.18	82.70	94.46	98.10	96.90	99.70	97.91
$p(\lvert\hat{\tau}-\tau\rvert \le 3)$	49.95	88.78	89.65	96.86	99.30	97.80	99.70	98.47
$p(\lvert\hat{\tau}-\tau\rvert \le 4)$	57.55	92.34	93.75	97.84	99.70	98.23	99.70	98.73
$p(\lvert\hat{\tau}-\tau\rvert \le 5)$	63.25	94.67	95.80	98.28	99.90	98.45	100	98.85
Shift	*($1.5\sigma_1$, 0)*		*($1.75\sigma_1$, 0)*		*($2\sigma_1$, 0)*		*($2.5\sigma_1$, 0)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\lvert\hat{\tau}-\tau\rvert)$	0.030	0.282	0.025	0.122	0.022	0.086	0.027	0.004
$Std(\lvert\hat{\tau}-\tau\rvert)$	0.246	0.032	0.276	0.018	0.227	0.020	0.264	0.001
$p(\lvert\hat{\tau}-\tau\rvert = 0)$	97.60	93.76	98.30	96.58	98.40	98.17	98.15	99.70
$p(\lvert\hat{\tau}-\tau\rvert \le 1)$	99.80	97.83	99.85	98.95	99.85	99.47	99.75	99.95
$p(\lvert\hat{\tau}-\tau\rvert \le 2)$	99.85	98.60	99.85	99.35	99.85	99.72	99.75	99.97
$p(\lvert\hat{\tau}-\tau\rvert \le 3)$	99.85	98.90	99.85	99.49	99.85	99.76	99.80	100
$p(\lvert\hat{\tau}-\tau\rvert \le 4)$	99.85	99.17	99.85	99.58	99.85	99.80	99.80	100
$p(\lvert\hat{\tau}-\tau\rvert \le 5)$	100	99.29	99.90	99.61	100	99.81	100	100

Table 2 Comparison between neural network B and MLE approach under different shifts in the mean of second quality characteristic

Shift	*(0, 0.5σ₂)*		*(0, 0.75σ₂)*		*(0, σ₂)*		*(0, 1.25σ₂)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\lvert\hat{\tau}-\tau\rvert)$	7.342	1.472	1.245	0.790	0.160	0.548	0.030	0.369
$Std(\lvert\hat{\tau}-\tau\rvert)$	7.588	0.047	2.071	0.045	0.578	0.042	0.294	0.039
$p(\lvert\hat{\tau}-\tau\rvert = 0)$	13.40	50.21	51.95	72.78	88.13	85.70	97.98	92.35
$p(\lvert\hat{\tau}-\tau\rvert \le 1)$	23.70	74.40	71.28	91.15	97.63	95.17	99.73	97.31
$p(\lvert\hat{\tau}-\tau\rvert \le 2)$	33.06	84.68	83.68	95.67	99.43	97.18	99.83	98.29
$p(\lvert\hat{\tau}-\tau\rvert \le 3)$	41.00	90.62	90.48	97.31	99.75	97.85	99.88	98.63
$p(\lvert\hat{\tau}-\tau\rvert \le 4)$	47.94	93.83	94.00	97.92	99.78	98.28	99.88	98.88
$p(\lvert\hat{\tau}-\tau\rvert \le 5)$	53.72	95.64	95.43	98.32	99.78	98.58	99.88	99.06
Shift	*(0, 1.5σ₂)*		*(0, 1.75σ₂)*		*(0, 2σ₂)*		*(0, 2.5σ₂)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\lvert\hat{\tau}-\tau\rvert)$	0.029	0.163	0.031	0.044	0.039	0.086	0.032	0.001
$Std(\lvert\hat{\tau}-\tau\rvert)$	0.341	0.025	0.344	0.009	0.456	0.020	0.397	0.001
$p(\lvert\hat{\tau}-\tau\rvert = 0)$	98.50	96.39	98.25	98.45	98.53	98.17	98.63	99.95
$p(\lvert\hat{\tau}-\tau\rvert \le 1)$	99.68	98.83	99.70	99.48	99.55	99.47	99.68	1
$p(\lvert\hat{\tau}-\tau\rvert \le 2)$	99.75	99.26	99.75	99.70	99.58	99.72	99.70	1
$p(\lvert\hat{\tau}-\tau\rvert \le 3)$	99.80	99.40	99.80	99.79	99.60	99.76	99.70	1
$p(\lvert\hat{\tau}-\tau\rvert \le 4)$	99.80	99.50	99.80	99.84	99.60	99.80	99.70	1
$p(\lvert\hat{\tau}-\tau\rvert \le 5)$	99.80	99.58	99.80	99.87	99.60	99.81	99.70	1

By the way, we understand that performance of the extended MLE approach under shifts in the Normal quality characteristic is roughly better than Poisson quality characteristic by comparison results of Tables 1 and 2. It is obvious that the performance of both proposed estimators are sufficient under different separate and simultaneous shifts while the small, moderate and large shifts in the mean vector of the process are considered.

We can conclude form Tables 1 to 3 that in most situations including separate and simultaneous mean shifts the proposed ANN-based approach outperforms the extended MLE approach. Moreover, despite of MLE, the ANN-based approach does not require any transformation on the distribution of process data.

Table 3 Comparison between neural network C and MLE approach under different shifts in the mean of both quality characteristics

shift	*(0.5σ₁, 0.5σ₂)*		*(0.75σ₁, 0.75σ₂)*		*(σ₁, σ₂)*		*(1.25σ₁, 1.25σ₂)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\|\hat{\tau}-\tau\|)$	5.624	1.249	1.078	0.673	0.126	0.415	0.024	0.306
$Std(\|\hat{\tau}-\tau\|)$	6.099	0.048	1.782	0.044	0.514	0.037	0.266	0.035
$p(\|\hat{\tau}-\tau\|=0)$	17.60	55.75	56.24	77.78	91.08	88.67	98.30	93.96
$p(\|\hat{\tau}-\tau\|\le 1)$	29.86	79.21	74.82	93.14	98.06	96.29	99.78	97.94
$p(\|\hat{\tau}-\tau\|\le 2)$	39.76	88.64	84.24	96.38	99.18	97.92	99.90	98.71
$p(\|\hat{\tau}-\tau\|\le 3)$	48.54	93.41	90.60	97.52	99.62	98.41	99.90	98.95
$p(\|\hat{\tau}-\tau\|\le 4)$	55.76	95.79	94.54	98.12	99.80	98.71	99.90	99.09
$p(\|\hat{\tau}-\tau\|\le 5)$	62.36	97.10	96.86	98.40	99.82	98.91	99.90	99.21
shift	*(1.5σ₁, 1.5σ₂)*		*(1.75σ₁, 1.75σ₂)*		*(2σ₁, 2σ₂)*		*(0, 2.5σ₂)*	
	ANN	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*	*ANN*	*MLE*
$E(\|\hat{\tau}-\tau\|)$	0.012	0.109	0.013	0.02	0.012	0.004	0.012	0.001
$Std(\|\hat{\tau}-\tau\|)$	0.145	0.018	0.169	0.007	0.113	0.001	0.154	0.001
$p(\|\hat{\tau}-\tau\|=0)$	98.96	97.39	99.00	99.83	98.90	99.70	98.98	99.98
$p(\|\hat{\tau}-\tau\|\le 1)$	99.94	98.99	99.92	99.83	99.94	99.93	99.94	1
$p(\|\hat{\tau}-\tau\|\le 2)$	99.98	99.40	99.96	99.93	1.00	99.97	99.98	1
$p(\|\hat{\tau}-\tau\|\le 3)$	99.98	99.61	99.96	99.96	1.00	99.99	99.98	1
$p(\|\hat{\tau}-\tau\|\le 4)$	99.98	99.66	99.96	99.98	1.00	99.99	99.98	1
$p(\|\hat{\tau}-\tau\|\le 5)$	99.98	99.69	99.98	99.99	1.00	1.00	99.98	1

6 An illustrative example

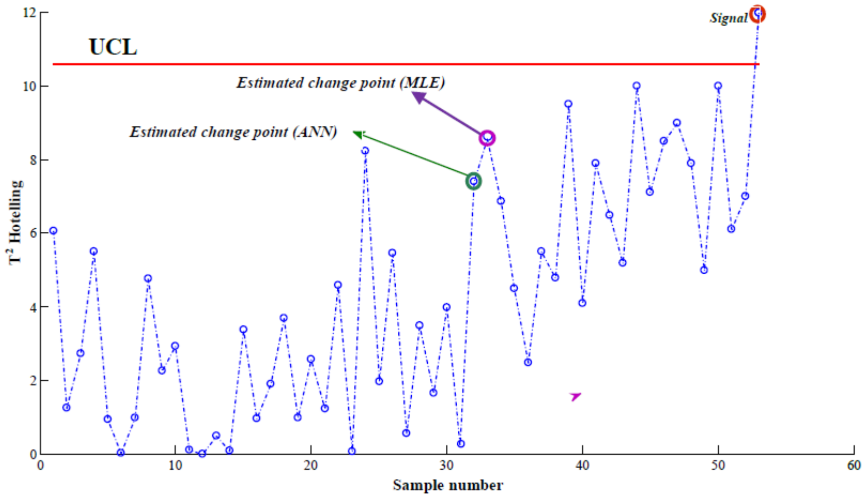
In order to demonstrate the application of the proposed methods, an illustrative example using the data of the last section is given in this section. Also, a comparison between proposed change point estimators is conducted in which the T^2 statistics corresponding to the random samples are shown graphically. For this purpose, consider an out-of-control situation where a step shift with the magnitude of σ_1 in the mean value of x_1 is occurred. The run length value obtained by the neural network in the first module is equal to 8. Consequently, we generate 32 in-control samples and calculate the corresponding input vectors. After that, we enter the 32 input vectors to the neural network 1 and record the values of the first output. We consider these output values as the 32 first elements of the input vector of neural network A in module 2. After that, we generate eight out-of-control samples with corresponding shift magnitude. The first, output values of neural network of module 1 corresponding to these out-of-control samples are then calculated. Finally, these output values are devoted to the 33th, 34th, …, 40th elements of input vector of neural network A in module 2. Table 4 represents the inputs and the outputs of the first neural network in module 2:

Table 4 The inputs and outputs of ANN change point estimator for simulated dataset

Subgroup	*First output of ANN 1*	*Output of module 2*	*Target value*	*Subgroup*	*First output of ANN 1*	*Output of module 2*	*Target value*
1	0.0924	0.0487	0	21	–0.0103	–0.0940	0
2	0.1108	0.0315	0	22	–0.0061	0.2153	0
3	0.0073	0.0704	0	23	–0.0142	–0.1502	0
4	–0.0374	–0.1701	0	24	0.2167	0.0785	0
5	0.0259	0.0019	0	25	–0.0160	–0.1585	0
6	–0.0080	–0.0258	0	26	0.0225	0.2306	0
7	–0.0068	0.2825	0	27	0.0080	–0.0227	0
8	–0.0116	–0.0485	0	28	0.1489	–0.0090	0
9	–0.0003	0.0580	0	29	0.0475	0.0185	0
10	–0.0060	–0.0313	0	30	0.0326	–0.0590	0
11	0.0367	–0.0454	0	31	0.0034	–0.3613	0
12	–0.0068	–0.3209	0	32	0.0069	0.1089	0
13	–0.0452	–0.0773	0	33	1.0024	0.7708	1
14	0.0214	0.0459	0	34	0.3483	0.1197	0
15	0.0023	–0.0248	0	35	0.6263	0.0190	0
16	–0.0183	–0.0784	0	36	0.9986	–0.1071	0
17	–0.0476	0.1606	0	37	0.0034	–0.1281	0
18	–0.0013	0.0671	0	38	0.8487	0.1903	0
19	–0.0555	0.1882	0	39	0.9931	0.5880	0
20	0.0062	–0.1600	0	40	0.3519	0.2218	0

As shown in Table 4, for the first 32 samples, the numbers in the first column (first output of ANN I) are close to zero whereas for the last eight samples these numbers are close to one. Moreover, the 33th output element obtained from neural network A in module 2 has the maximum value. Hence, the time of change point by the proposed neural network is equal to $\hat{\tau} = 32$ (The number of output node with the maximum value minus one). After that, the same data of the ANN is also used in MLE approach. As shown in Figure 2, by extended MLE approach the change point is estimated on 33th sample while it is estimated on 32th sample using the proposed ANN methodology. In other words, the proposed ANN-based change point estimator performs more accurate than the MLE approach.

Figure 3 Comparison of proposed estimators through the simulated data (see online version for colours)



7 Conclusions and future researches

In this paper a modular neural network-based approach was proposed to estimate the time of mean changes in the multivariate-attribute processes. In the first module, using a three-layer perceptron neural network in the literature, we detected mean shifts and diagnosed the sources of variation in the mean vector of the multivariate-attribute process. Then, based on the results of module 1, a model which in turn consists of modular neural networks for estimating change point in the process mean was suggested. We also extended an MLE approach and then compared the results of the proposed methods under different out-of-control scenarios. The results of the simulation studies showed the high performance of both methods under different separate and simultaneous mean shifts with different magnitudes. The results also represented that in most situations the proposed modular neural network-based model outperforms the extended MLE approach. As a future research, it is recommended to extend these methods to find the real time of a linear drift or monotonic change in the mean vector of multivariate-attribute processes.

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References

Ahmadzadeh, F. and Noorossana, R. (2008) 'Identifying the time of a step change with MEWMA control charts by artificial neural network', *The 8th International Industrial Engineering and Engineering Management Conference*, Singapore.

Ahmadzadeh, F., Lundberg, J. and Stromberg, T. (2011) 'Multivariate process parameter change identification by neural network', *The International Journal of Advanced Manufacturing Technology*, Vol. 69, Nos. 9–12, pp.2261–2268.

Amiri, A. and Allahyari, S. (2012) 'Change point estimation methods for control chart postsignal diagnostics: a literature review', *Quality and Reliability Engineering International*, Vol. 28, No. 7, pp.673–685.

Amiri, A., Maleki, M.R. and Doroudyan, M.H. (2015) 'Monitoring variability of multivariate-attribute processes using artificial neural network', *Production & Operations Management*, Vol. 5, No. 2, pp.21–36.

Atashgar, K. and Noorossana, R. (2010) 'Identifying change point in a bivariate normal process mean vector with monotonic changes', *The International Journal Industrial and System Engineering and Production Management*, Vol. 21, No. 1, pp.1–13.

Atashgar, K. and Noorossana, R. (2012) 'Diagnosing the source(s) of a monotonic change in the process mean vector', *The International Journal Advanced Manufacturing Technology*, Vol. 60, Nos. 9–12, pp.1175–1183.

Chen, L.H. and Wang, T.Y. (2004) 'Artificial neural networks to classify mean shifts from multivariate χ^2 chart signals', *Computers & Industrial Engineering*, Vol. 47, Nos. 2–3, pp.195–205.

Cherubini, U., Luciano, E. and Vecchiato, W. (2004) *Copula Methods in Finance*, John Wiley & Sons, The Atrium, Southern Gate, Chichester, West Sussex, England.

Doğu, E. and Kocakoc, I.D. (2011) 'Estimation of change point in generalized variance control chart', *Communications in Statistics-Simulation and Computation*, Vol. 40, No. 3, pp.345–363.

Doğu, E. and Kocakoc, I.D. (2013) 'A multivariate change point detection procedure for monitoring mean and covariance simultaneously', *Communications in Statistics-Simulation and Computation*, Vol. 42, No. 6, pp.1235–1255.

Doroudyan, M.H. and Amiri, A. (2013) 'Monitoring multivariate–attribute processes based on transformation techniques', *The International Journal of Advanced Manufacturing Technology*, Vol. 69, Nos. 9–12, pp.2161–2172.

Doroudyan, M.H. and Amiri, A. (2014) 'Monitoring correlated variable and attribute quality characteristics based on NORTA inverse technique', *International Journal of Productivity and Quality Management*, Vol. 14, No. 2, pp.247–262.

Hou, C.D., Shao, Y.E. and Huang, S. (2011) 'Estimation of the change point of a multivariate normal process using the |S| control chart and MLE method', *ICIC Express Letters – An International Journal of Research and Surveys*, Vol. 5, No. 6, pp.1845–1849.

Kazemzadeh, R.B., Noorossana, R. and Ayoubi, M. (2015) 'Change point estimation of multivariate linear profiles under linear drift', *Communications in Statistics-Simulation and Computation*, Vol. 44, No. 6, pp.1570–1599.

Li, F., Runger, G.C. and Tuv, E. (2006) 'Supervised learning for change-point detection', *The International Journal of Production Research*, Vol. 44, No. 14, pp.2853–2868.

Maleki, M.R. and Amiri, A. (2015) 'Simultaneous monitoring of multivariate-attribute process mean and variability using artificial neural networks', *Journal of Quality Engineering and Production Optimization*, Vol. 1, No. 1, pp.44–54.

Maleki, M.R. and Sahraeian, R. (2015) 'Online Monitoring and fault diagnosis of multivariate-attribute process mean using neural networks and discriminant analysis technique', *International Journal of Engineering-Transactions B: Applications*, Vol. 28, No. 11, pp.1634–1643.

Maleki, M.R., Amiri, A. and Doroudyan, M.H. (2012) 'Monitoring multivariate-attribute processes using artificial neural network', *Proceedings of 42th Conference on Computers and Industrial Engineering (CIE42)*, pp.135–144.

Maleki, M.R., Amiri, A. and Mousavi, S.M. (2015) 'Step change point estimation in the multivariate-attribute process variability using artificial neural networks and maximum likelihood estimation', *Journal of Industrial Engineering International*, Vol. 11, No. 4, pp.505–515.

Maleki, M.R., Mousavi, S.M. and Amiri, A. (2015) 'A new neural network-based control scheme for fault detection and fault diagnosis in fuzzy multivariate multinomial data', *International Journal of Applied Decision Sciences*, Vol. 8, No. 2, pp.127–144.

Matteson, D.S. and James, N.A. (2014) 'A nonparametric approach for multiple change point analysis of multivariate data', *Journal of the American Statistical Association*, Vol. 109, No. 505, pp.334–345.

Nedumaran, G., Pignatiello Jr, J.J. and Calvin, J.A. (2000) 'Identifying the time of a step-change with χ^2 control charts', *Quality Engineering*, Vol. 13, No. 2, pp.153–159.

Niaki, S.A. and Abbasi, B. (2008a) 'Detection and classification mean-shifts in multi-attribute processes by artificial neural networks', *International Journal of Production Research*, Vol. 46, No. 11, pp.2945–2963.

Niaki, S.T.A. and Abbasi, B. (2005) 'Fault diagnosis in multivariate control charts using artificial neural networks', *Quality and Reliability Engineering International*, Vol. 21, No. 8, pp.825–840.

Niaki, S.T.A. and Abbasi, B. (2007a) 'Bootstrap method approach in designing multi-attribute control charts', *The International Journal of Advanced Manufacturing Technology*, Vol. 35, Nos. 5–6, pp.434–442.

Niaki, S.T.A. and Abbasi, B. (2007b) 'Skewness reduction approach in multi-attribute process monitoring', *Communications in Statistics – Theory and Methods*, Vol. 36, No. 12, pp.2313–2325.

Niaki, S.T.A. and Abbasi, B. (2008b) 'Generating correlation matrices for normal random vectors in NORTA algorithm using artificial neural networks', *Journal of Uncertainty System*, Vol. 2, No. 3, pp.192–201.

Niaki, S.T.A. and Khedmati, M. (2014) 'Step change-point estimation of multivariate binomial processes', *The International Journal of Quality Science*, Vol. 31, No. 5, pp.566–587.

Rezazadeh, N.M. (2014) *Multi-Variate-Attribute Quality Control (MVAQC)*, PhD thesis, The University of Melbourne.

Samuel, T.R., Pignatiello, J.J. and Calvin, J.A. (1998) 'Identifying the time of a step change with X control charts', *Journal of Quality Engineering*, Vol. 10, No. 3, pp.521–527.

Shao, Y.E. and Hou, C.D. (2013) 'Change point determination for a multivariate process using a two-stage hybrid scheme', *Applied Soft Computing*, Vol. 13, No. 3, pp.1520–1527.

Zamba, K.D. and Hawkins, D.M.A. (2006) 'Multivariate change-point model for statistical process control', *Technometrics*, Vol. 48, No. 4, pp.539–549.

Zarandi, M.H.F. and Alaeddini, A. (2010) 'A general fuzzy-statistical clustering approach for estimating the time of change in variable sampling control charts', *Information Sciences*, Vol. 180, No. 16, pp.3033–3044.