Decreasing the effect of measurement errors on detecting and diagnosing performance of MAX-EWMAMS control chart in Phase II

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Abstract: In recent years, the quality practitioners have concentrated on exploring the effect of measurement errors on the performance of control charts. To the best of our knowledge, the effect of measurement errors on simultaneous monitoring of the process mean and the process variability is neglected in the literature. In this paper, the effect of measurement errors on detecting and diagnosing performance of one of the most common simultaneous monitoring approaches in the literature is investigated. Two approaches namely multiple measurement approach as well as increasing sample size are suggested for compensating for the effect of measurement errors. The results of simulation study show that the measurement errors can adversely affect the detecting performance of maximum exponentially weighted moving average and mean squared deviation (MAX-EWMAMS) control chart while the effect of measurement errors on diagnosing performance of this control chart is negligible. The results also represent that both tasking multiple measurement on each sample point and increasing sample size can adequately compensate for the measurement errors effect.

Keywords: measurement errors; MAX-EWMAMS control chart; statistical process monitoring; SPM; Phase II; simultaneous monitoring.

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1 Introduction

Control charts are one of the most efficient tools in statistical process monitoring (SPM) which are used for online assessment of manufacturing or service processes. The main objective of control charts is to detect any change in the process parameters. Various control charts available in the literature have been constructed by assuming that the inspected samples are accurate. However, in most quality monitoring applications, a difference between the actual and observed quantity may be present due to measuring instruments errors. Theses measurement errors may affect the performance of different control charts. Hence, the measurement errors must be considered in constructing control charts for online monitoring of process parameters. The effect of measurement errors on the performance of various monitoring schemes is investigated by some researchers. In one of the earliest works on this problem, Bennet (1954) explored the effect of measurement errors on $\bar{X}$ control chart. Bennet (1954) used the model $Y = X + \varepsilon$ where X and Y are the true and the observed values of quality characteristic under investigation, respectively while ε is the random error due to the measurement error. Then, Kanazuka (1986) extended Bennet's (1954) work by studying effect of measurement error on $\bar{X} / R$ control chart. The effect of measurement errors on various control charts is also addressed by Linna et al. (2001) (on χ^2 chart), Yang and Yang (2005) (on Shewhart and cause-selecting charts), Yang et al. (2007) [on exponentially weighted moving average (EWMA) and cause-selecting charts], Maravelakis (2007) (on cumulative sum chart), Moameni et al. (2012) (on $\tilde{X} - \tilde{R}$ fuzzy charts), Khurshid and Chakraborty (2014) (on zero-truncated binomial distribution-based chart), Abbasi (2014) (on Shewhart, EWMA and cumulative sum charts) and Noorossana and Zerehsaz (2015) (on charts for monitoring simple linear profiles). For more information about the effect of gauge measurement errors on the performance of different control charts, please refer review paper provided by Maleki et al. (2016). Recently, some remedial approaches have been introduced to compensate for the effect of measurement errors on the performance of control charts in some researches such as Maravelakis et al. (2004), Cocchi and Scagliarini (2007), Abbasi (2010), Costa and Castagliola (2011), Maravelakis (2012), Hu et al. (2015), Riaz (2014), Haq et al. (2015), Hu et al. (2015), and Abbasi (2016).

In recent years, simultaneous monitoring of process mean and variability by a single statistic has been taken into account by the researchers. In simultaneous monitoring, the main idea is to use a single statistic to monitor the process mean and variability, jointly. Some of the recent works in the simultaneous monitoring area includes Tasias and Nenes (2012), Sheu et al. (2013), Chowdhury et al. (2015) and Maleki and Amiri (2015).

All of the above-mentioned researches in measurement error area have been focused on exploring the effect of measurement errors on monitoring the process mean or the process variability, separately. However, the effect of measurement errors on simultaneous monitoring of process mean and variability is neglected in the literature. In this paper the effect of measurement errors on the performance of MAX-EWMAMS control chart to detect simultaneous changes in the parameters of the process mean and variability is explored. Moreover, we show that the effect of measurement errors on the performance of this chart to diagnose the source of signal (mean, variability or both) is negligible. We also utilise the multiple measurements on each sample point to decrease the effect of measurement errors on both detecting and diagnosing performance of MAX-EWMAMS control chart.

The rest of this paper is organised as follows: in Section 2, we discuss MAX-EWMAMS control chart. In Section 3, modification of MAX-EWMAMS control chart in the presence of measurement error is proposed. In Section 4, the multiple measurement approach is developed to compensate the effect of measurement error on the performance of MAX-EWMAMS control chart. In Section 5, simulation studies are given to investigate the effect of measurement errors on detecting and diagnosing performance of MAX-EWMAMS control chart and to show the efficiency of the multiple measurement approach. In Section 6, the conclusions and a recommendation for future study are given.

2 MAX-EWMAMS control chart

Let $\mathbf{X}_t = [X_{t1}, \ldots, X_{tn}]^T$ denote the observations of quality characteristic under investigation at t^{th}; $t = 1, 2, \ldots$ sample point. We assume that when the process is in-control, X_{tj}; $j = 1, \ldots, n$ follows a normal distribution with parameters μ_0 and σ_0^2.. The EWMA statistic for monitoring the process mean is:

$$Z_t = \lambda \bar{X}_t + (1-\lambda) Z_{t-1}, \tag{1}$$

where $\bar{X}_t = \frac{\sum_{j=1}^{n} X_{tj}}{n}$, λ is the smoothing parameter which can be selected in the range of [0, 1] and $Z_0 = \mu_0$. At tth sample point, the EWMS statistic which is first proposed by MacGregor and Harris (1993) for monitoring the process variability will be:

$$S_t^2 = (1-\lambda)S_{t-1}^2 + \lambda\sum_{j=1}^{n}\frac{\left(X_{tj}-\mu_0\right)^2}{n}, \tag{2}$$

where $S_0^2 = \sigma_0^2$. It can be statistically checked that:

$$E\left[S_t^2\right] = \sigma_0^2, \tag{3}$$

$$Var\left[S_t^2\right] = \frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma_0^2. \tag{4}$$

In order to have a single statistic for simultaneous monitoring of the process mean and variability, the EWMA and EWMS statistics are transformed to the standardised normal distribution. It can be statistically checked that the distribution of Z_t in equation (1) is:

$$N\left(\mu_0, \frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma_0^2\right. \tag{5}$$

Obviously, the standardised statistic for monitoring the process mean is:

$$U_t = \frac{\left(Z_t-\mu_0\right)}{\sqrt{\frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma_0^2}}. \tag{6}$$

It can be shown that in situations where the observations are independent and normally distributed, for each value of sample size n, when $t \rightarrow \infty$, S_t^2/σ_0^2 approximately follows a chi-square distribution with the degree of freedom equal to $df = n(2 - \lambda) / \lambda$. Therefore, the following transformation is utilised to have a control statistic for monitoring the process variability with approximately standardised normal distribution.

$$V_t = \phi^{-1}\left[\left(\chi_{df}^2 \le \frac{df \times S_t^2}{\sigma_0^2}\right)\right], \tag{7}$$

Finally, the MAX-EWMAMS statistic at sample point t; $t = 1, 2, \ldots$ is defined as follows:

$$M_t = \max\left\{\left|U_t\right|, \left|V_t\right|\right\}. \tag{8}$$

Since $M_t \geq 0$, the MAX-EWMAMS control chart only has upper control limit (*UCL*). As a consequence, the MAX-EWMAMS control chart triggers an out-of-control signal if $M_t > UCL$, where *UCL* is set to have a pre-specified in-control average run length (ARL_0).

3 Modification of MAX-EWMAMS control chart in the presence of measurement error

In the presence of measurement error, the true value of quality characteristic X is not directly observable, but can only be assessed by the results of the measured quantities. In this paper, the additive covariate model is used to describe the relationship between true and measured quantities as follows:

$$Y = A + BX + \varepsilon, \tag{9}$$

where A and B are the known constants while ε is the error term which is independent from X and follows a normal distribution with mean zero and constant variance of σ_ε^2. Therefore we have $Y \sim N\left(A + B\mu_0, B^2\sigma_0^2 + \sigma_\varepsilon^2\right)$.. The EWMA statistic for monitoring the process mean is:

$$Z_t = \lambda \overline{Y}_t + (1-\lambda) Z_{t-1}, \tag{10}$$

where $Z_0 = A + B\mu_0$ and $\overline{Y}_t = \frac{\sum_{j=1}^{n} Y_{tj}}{n}$. For monitoring the process variability in the presence of measurement error, the EWMS statistic is rewritten as:

$$S_t'^2 = (1-\lambda) S_{t-1}'^2 + \lambda \sum_{j=1}^{n} \frac{\left(Y_{tj} - \left(A + B\mu_0\right)\right)^2}{n}, \tag{11}$$

where $S_0^2 = B^2\sigma_0^2 + \sigma_\varepsilon^2$. It would be statistically checked that:

$$Z_t \sim N\left(A + B\mu_0, \left(\frac{\lambda}{n(2-\lambda)}\right)\left[1-(1-\lambda)^{2t}\right] \times \left(B^2\sigma_0^2 + \sigma_\varepsilon^2\right)\right) \tag{12}$$

Hence, in the presence of measurement error, the standardised EWMA-based statistic for monitoring the process mean is defined as:

$$U_t' = \frac{Z_t - \left(A + B\mu_0\right)}{\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right] \times \frac{B^2\sigma_0^2 + \sigma_\varepsilon^2}{n}}} \tag{13}$$

In order to obtain the standardised statistic for monitoring the process variability, equation (7) is rewritten as follows:

$$V_t' = \phi^{-1}\left[\Pr\left(\chi_{df}^2 \le \frac{df \times S_t'^2}{\sigma_y^2}\right)\right]. \tag{14}$$

Finally, the chart statistic considering the measurement error is obtained as follows:

$$M_t' = \max\left\{\left|U_t'\right|, \left|V_t'\right|\right\} \tag{15}$$

The chart triggers an out-of-control signal if $M_t' > UCL$, where UCL is set to have a pre-specified ARL_0.

As long as $M_t' > UCL$, the following post-signal diagnostic rules are implemented to identify the source of the signal (mean or variability):

1 if $|U_t'| > UCL$ and $\left|V_t'\right| UCL$, the deviation in the process mean is considered as the source of the signal

2 if $|V_t'| UCL$ and $|U_t'| > UCL,$ the deviation in the process variability is considered as the source of the signal

3 if $|U_t'| > UCL$ and $|V_t'| UCL,$ the deviation in both process mean and variability is considered as the source of the signal.

Assume that a given simultaneous shift ($\mu_X = \mu_0 + \delta\sigma_0$, $\sigma_X = \psi\sigma_0$) leads to an out-of-control signal at t^{th} sample. In such situation, the deviation in both mean and variance of the process must be considered as the source of signal. However, the performance of MAX-EWMAMS control chart in diagnosing the source of signal under simultaneous shifts is not satisfactory. Because the probability that both $|U_t'|$ or $|V_t'|$ statistics jointly exceed the *UCL* $\left(|P\left(U_t' > UCL, V_t' > UCL \middle| \mu_X, \sigma_X\right)\right)$ is less than the probability that only one of them exceeds the *UCL*

$$\left(P\left(|U'|_t > UCL, |V_t'| \leq UCL \middle| \mu_X, \sigma_X\right) + P\left(|V_t'| > UCL, |U_t'| \leq UCL \mid \mu_X, \sigma_X\right)\right).$$

4 Multiple measurement approach

Linna and Woodall (2001) suggested that taking several in each sampled unit can decrease the effect of measurement error. In this section, we utilise this approach to compensate the measurement error effect on the performance of MAX-EWMAMS control chart. To incorporate the multiple measurement approach in our model, we define matrix for *t*th sample point as follows:

$$\underline{\mathbf{Y}}_t = \begin{bmatrix} Y_{t11} & Y_{t21} & \cdots & Y_{tn1} \\ Y_{t12} & Y_{t22} & \cdots & Y_{tn2} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{t1k} & Y_{t2k} & \cdots & Y_{tnk} \end{bmatrix}, \tag{16}$$

where Y_{tji}; $t = 1, 2, \ldots$ $j = 1, 2, \ldots, n$ $i = 1, 2, \ldots, k$ is the j^{th} observation of the measured quality characteristic at t^{th} sample point which is obtained by i^{th} measurement. For the average value of j^{th} observation at t^{th} sample point we have

$$\overline{Y}_{tj} = \frac{\sum_{i=1}^{k} Y_{tji}}{k} \sim N\left(A + B\mu_0, B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{k}\right).$$

The modified EWMA control statistic for monitoring the process mean is defined as:

$$Z_t'' = \lambda \overline{\overline{Y}}_t + (1-\lambda) Z_{t-1}'', \tag{17}$$

where $Z_0'' = A + B\mu_0$ and $\overline{\overline{Y}}_t = \frac{\sum_{k=1}^{n} \overline{Y}_{tj}}{n}$ is the overall mean at t^{th} sample point with the following distribution:

$$\bar{\bar{Y}}_t \sim N\left(A + B\mu_0, \frac{B^2\sigma_0^2}{n} + \frac{\sigma_\varepsilon^2}{nk}\right) \tag{18}$$

The control statistic for monitoring the process variability will be:

$$S_t''^2 = (1-\lambda)S_{t-1}''^2 + \lambda\sum_{j=1}^{n}\frac{\left(\bar{Y}_{tj} - \left(A + B\mu_0\right)\right)^2}{n}, \tag{19}$$

where $S_0^2 = B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{k}$. Since $Z_t'' \sim N\left(A + B\mu_0, \frac{\lambda}{(2-\lambda)}\left(1-(1-\lambda)^{2t}\right)\left(\frac{B^2\sigma_0^2}{n} + \frac{\sigma_\varepsilon^2}{nk}\right)\right)$, the standardised control statistic for monitoring is defined as:

$$U_t'' = \frac{\left[Z_t'' - (A + B\mu_0)\right]}{\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\times\left(\frac{B^2\sigma_0^2}{n} + \frac{\sigma_\varepsilon^2}{nk}\right)}}. \tag{20}$$

Utilising multiple measurement approach, leads to the following standardised statistic for monitoring the process variability:

$$V_t'' = \phi^{-1}\left[\Pr\left(\chi_{df}^2 \le \frac{df \times S_t''^2}{\sigma_{\bar{Y}}^2}\right)\right], \tag{21}$$

where $\sigma_{\bar{Y}}^2 = B^2\sigma_0^2 + \frac{\sigma_\varepsilon^2}{k}$.. Finally, the joint monitoring statistic in the presence of measurement error and utilising multiple measurement approach is obtained as:

$$M_t'' = \max\left\{\left|U_t''\right|, \left|V_t''\right|\right\} \tag{22}$$

The chart triggers an out-of-control signal if $M_t' > UCL$, where UCL is set to have a pre-specified ARL_0.

5 Performance evaluation

The run length is defined as the number of consecutive samples which is taken and plotted until the first statistic falls outside the control limits interval. The ARL is the most common criterion to evaluate the detecting performance of different monitoring schemes in the SPM literature. In this study, the ARL along with the standard deviation of run lengths (SDRL) under different out-of-control step changes is used to evaluate the detecting performance of the proposed monitoring schemes. In order to provide the ARLs and SDRLs under out-of-control scenarios, the value of UCL for each scenario is set by simulation experiments such that the in-control ARL (ARL_0) becomes approximately equals to 200. Then, the out-of-control ARLs and SDRLs are obtained based on 20,000 simulation replicates in MATLAB software. In the out-of-control scenarios, the step mean and variance changes are denoted by $\mu = \mu_0 + \delta\sigma_0$ and $\sigma = \psi\sigma_0$, respectively. Here we assume that when the process is in-control, X follows a standard normal distribution.

Without loss of generality, we also assume the parameters of the additive covariate model to be as $A = 0$ and $B = 1$. Table 1 represents the ARLs and SDRLs for different value of σ_ε^2 when $n = 5$ and $\lambda = 0.1$. The results of Table 1 confirm the adverse effect of measurement errors on detecting performance of MAX-EWMAMS control chart under all mean shifts, variance shifts and simultaneous shifts. We can see that, as the variance of measurement error increases, the ARLs and SDRLs tend to be increased. Moreover, the effect of measurement errors on detecting performance of MAX-EWMAMS control chart lessens as the magnitude of step shift increases. The diagnosing performance of the MAX-EWMAMS control chart in terms of correct diagnosis percentage (CDP) when $n = 5$ and $\lambda = 0.1$ is summarised in Table 2. Table 2 reveals that the effect of measurement error on diagnosing performance of MAX-EWMAMS is negligible.

Table 1 ARLs and SDRLs for different values of σ_ε^2 when $n = 5$, $\lambda = 0.1$, $k = 1$

UCL		*2.7247*	*2.7247*	*2.7247*	*2.7247*	*2.7247*
(δ, ψ)	σ_ε^2	*No error*	*0.25*	*0.5*	*0.75*	*1*
(0, 0)	ARL	199.6433	203.2319	199.7815	203.0724	204.4138
	SDRL	201.4097	196.4947	198.4662	200.9392	202.4140
(0.25, 1)	ARL	20.8978	25.0083	29.7384	33.1850	37.4397
	SDRL	16.1371	19.7284	24.2172	27.8107	32.0677
(0.5, 1)	ARL	6.3182	7.5904	8.9602	10.1408	11.3432
	SDRL	3.8958	4.8949	5.9357	6.8166	7.9706
(0.75, 1)	ARL	3.3099	3.9129	4.5643	5.1586	5.7625
	SDRL	1.8838	2.2631	2.7112	3.1469	3.5377
(0, 0.5)	ARL	6.5273	8.9189	11.8481	15.2974	19.5017
	SDRL	0.7946	1.9792	3.7710	6.1885	9.4458
(0, 1.1)	ARL	45.4458	59.5822	73.3957	85.6144	97.4451
	SDRL	40.8953	54.8862	68.3600	82.4126	94.1649
(0, 1.25)	ARL	12.3347	12.9500	21.0773	25.8005	30.9053
	SDRL	8.8364	10.5108	16.9313	21.1270	26.7778
(0.25, 0.5)	ARL	7.2975	9.8088	12.6269	16.6616	19.0256
	SDRL	1.0959	2.4332	4.2328	6.5428	9.3370
(0.25, 1.1)	ARL	16.4323	20.1841	23.9096	27.4807	30.9309
	SDRL	12.7467	16.1232	19.5265	23.1843	26.3321
(0.25, 1.25)	ARL	9.3211	11.7683	14.3351	17.2021	19.7014
	SDRL	6.5808	8.6129	10.9543	13.3625	15.8247
(0.5, 0.5)	ARL	6.3258	7.6130	8.7692	10.0639	11.2585
	SDRL	1.7705	2.6833	3.6136	4.6480	5.6481
(0.5, 1.1)	ARL	6.1194	7.3289	8.5693	9.6873	10.7659
	SDRL	4.0991	4.9756	5.9145	6.9189	7.5642
(0.5, 1.25)	ARL	5.2568	6.3487	7.4277	8.5005	9.5704
	SDRL	3.4933	4.3032	5.1212	6.0615	6.7699

Table 2 CDP for different values of σ_ε^2 when n = 5, λ = 0.1, k = 1

(δ, ψ)	*Signal factor* \ σ_ε^2	*No error*	*0.25*	*0.5*	*0.75*	*1*
(0.25, 1)	Correct diagnosis percent	95.28	94.47	93.25	92.57	91.95
(0.5, 1)	Correct diagnosis percent	98.02	97.63	97.39	97.34	97.17
(0.75, 1)	Correct diagnosis percent	97.70	97.73	97.77	98.15	97.93
(0, 0.5)	Correct diagnosis percent	100	99.99	99.90	99.73	99.21
(0, 1.1)	Correct diagnosis percent	76.35	71.38	68.36	66.09	63.90
(0,1.25)	Correct diagnosis percent	83.97	83.27	82.25	80.67	79.47

Table 3 ARLs and SDRLs under multiple measurement approach when n = 5, λ = 0.1 and $\sigma_\varepsilon^2 = 1$ for different value of k

UCL		*2.7247*	*2.7247*	*2.7247*	*2.7247*	*2.7247*
(δ, ψ)	*k*	*No error*	*1*	*2*	*5*	*10*
(0, 0)	ARL	199.6433	204.4138	201.4260	205.8066	199.8006
	SDRL	201.4097	202.4140	199.8302	203.4022	198.1943
(0.25, 1)	ARL	20.8978	37.4397	29.6591	24.2778	22.6044
	SDRL	16.1371	32.0677	24.0479	19.2255	18.0760
(0.5, 1)	ARL	6.3182	11.3432	8.8433	7.3825	6.8818
	SDRL	3.8958	7.9706	5.8654	4.7533	4.4171
(0.75, 1)	ARL	3.3099	5.7625	5.5939	3.8343	3.5359
	SDRL	1.8838	3.5377	2.7075	2.1830	1.9936
(0, 0.5)	ARL	6.5273	19.5017	11.8555	8.3906	7.4353
	SDRL	0.7946	9.4458	3.7796	1.6657	1.1990
(0, 1.1)	ARL	45.4458	97.4451	74.6865	58.3012	51.2615
	SDRL	40.8953	94.1649	71.1094	54.7418	47.8262
(0, 1.25)	ARL	12.3347	30.9053	21.2676	15.4656	13.8284
	SDRL	8.8364	26.7778	17.2412	11.5136	10.2706
(0.25, 0.5)	ARL	7.2975	19.0256	12.6265	9.2878	8.2518
	SDRL	1.0959	9.3370	4.2455	2.0990	1.5404
(0.25, 1.1)	ARL	16.4323	30.9309	23.5854	19.3987	17.8377
	SDRL	12.7467	26.3321	20.0680	15.5673	14.9271
(0.25, 1.25)	ARL	9.3211	19.7014	14.4089	11.2748	10.1468
	SDRL	6.5808	15.8247	10.9809	8.3025	7.3394
(0.5, 0.5)	ARL	6.3258	11.2585	8.7888	7.3403	6.773
	SDRL	1.7705	5.6481	3.6594	2.5192	2.1448
(0.5, 1.1)	ARL	6.1194	10.7659	8.6203	7.0384	6.5964
	SDRL	4.0991	7.5642	5.9721	4.6974	4.4121
(0.5, 1.25)	ARL	5.2568	9.5704	7.4262	6.1359	5.6742
	SDRL	3.4933	6.7699	5.1000	4.1414	3.8481

Table 4 CDP under multiple measurement approach when n = 5, λ = 0.1 and $\sigma_\varepsilon^2 = 1$ for different value of k

(δ, ψ)	*Signal factor* \ *k*	*No error*	*1*	*2*	*5*	*10*
(0.25, 1)	Correct diagnosis percent	95.28	91.95	93.32	94.55	95.08
(0.5, 1)	Correct diagnosis percent	98.02	97.17	97.54	97.63	97.68
(0.75, 1)	Correct diagnosis percent	97.70	97.93	97.95	97.71	97.56
(0, 0.5)	Correct diagnosis percent	100	99.21	99.84	99.99	100
(0, 1.1)	Correct diagnosis percent	76.35	63.90	69.50	74.03	74.82
(0, 1.25)	Correct diagnosis percent	83.97	79.47	82.90	82.97	83.64

Table 5 Effect of measurement error and multiple measurement approach

Effect \ *Area*	*Detecting*			
	Mean shift		*Variance shift*	
Measurement error (n = 5)	$\sigma_\varepsilon^2 = 0.5$	$\sigma_\varepsilon^2 = 1$	$\sigma_\varepsilon^2 = 0.5$	$\sigma_\varepsilon^2 = 1$
	28.9	101.6	295.3	1072.4
Multiple measurements ($\sigma_\varepsilon^2 = 1$)	$k = 2$	$k = 5$	$k = 2$	$k = 5$
	22.3	64.2	223.1	631.4
Effect \ *Area*	*Diagnosing*			
	Mean shift		*Variance shift*	
Measurement error (n = 5)	$\sigma_\varepsilon^2 = 0.5$	$\sigma_\varepsilon^2 = 1$	$\sigma_\varepsilon^2 = 0.5$	$\sigma_\varepsilon^2 = 1$
	1.5	3.9	22.3	58.6
Multiple measurements ($\sigma_\varepsilon^2 = 1$)	$k = 2$	$k = 5$	$k = 2$	$k = 5$
	0.7	2.3	14.5	38.5

Table 3 contains the values of ARL and SDRL obtained by utilising multiple measurement approach for different values of parameter k when n = 5, λ 0.1 and $\sigma_\varepsilon^2 = 1$. One can observe that, we can compensate the effect of measurement errors by taking multiple measurements on each observation. It is also concluded that as the parameter k increases, both ARL and SDRL values decrease. The effect of taking multiple measurements on diagnosing performance of MAX-EWMAMS control chart is displayed in Table 4. Table 4 shows that the effect of multiple measurement approach on diagnosing performance of MAX-EWMAMS control chart is not considerable. For example in step shift with magnitude of (δ, ψ) = (0.5, 1), taking 2, 5 and 10 measurements on each sample point increases the correct diagnosis of MAX-EWMAMS chart only about 0.37%, 0.46% and 0.51%, respectively.

The results of Tables 1–4 in terms of $\frac{SS_E}{N}$ criterion are also summarised in Table 5 where N is the number of step shifts considered (three for mean and three for variance shifts). The results show that for both mean and variance shifts, as the value of σ_ε^2 increases, the difference between the values of ARL and corrected diagnosis percentage from their similar ones in error-free case increase. The results also show that for both mean and variance shifts, by increasing the value of parameter k, the difference between the values of ARL and corrected diagnosis percentage from their similar ones in the case of $k = 1$ increases.

Here, the effect of parameter n on detecting and diagnosing performance of MAX-EWMAMS chart when $\sigma_\varepsilon^2 = 0.5$, $\lambda = 0.1$ and $k = 1$ are investigated and the results are summarised in Tables 6 and 7. The results of Table 6 confirm that the detecting performance of MAX-EWMAMS chart under measurement errors is seriously affected by the value of parameter n. As the value of parameter n increases, the ARLs and SDRLs tend to decrease. As seen in Table 7, the diagnosing performance of MAX-EWMAMS chart improves as the value of parameter n increases.

Table 6 ARLs and SDRLs for different values of n when $\sigma_\varepsilon^2 = 0.5$, $\lambda = 0.1$, $k = 1$

UCL		*2.7216*	*2.7247*	*2.7299*	*2.7357*	*2.7396*
(δ, ψ) \ *n*		*3*	*5*	*10*	*12*	*15*
(0, 0)	ARL	198.1780	199.7815	197.5956	199.9626	203.9558
	SDRL	195.7899	198.4662	192.9538	195.5071	205.5351
(0.25, 1)	ARL	44.0630	29.7384	16.3242	13.8250	11.3942
	SDRL	38.9207	24.2172	11.8610	10.0362	7.6817
(0.5, 1)	ARL	13.4841	8.9602	4.9901	4.3037	3.6950
	SDRL	9.5761	5.9357	2.9966	2.4993	2.0925
(0.75, 1)	ARL	6.7873	4.5643	2.6247	2.3454	2.0033
	SDRL	4.3547	2.7112	1.4153	1.1891	0.9952
(0, 0.5)	ARL	17.4653	11.8481	7.4629	6.7275	5.9104
	SDRL	6.9807	3.7710	1.7123	1.4776	1.1872
(0, 1.1)	ARL	90.7900	73.3957	50.7420	45.8784	40.4083
	SDRL	88.5107	68.3600	45.7138	40.9917	35.3443
(0, 1.25)	ARL	28.9788	21.0773	13.0120	11.5510	10.0312
	SDRL	25.2987	16.9313	8.9257	7.5449	5.9967
(0.25, 0.5)	ARL	19.0308	12.6269	7.7314	6.9633	6.0567
	SDRL	7.7759	4.2328	1.9809	1.7141	1.4539
(0.25, 1.1)	ARL	34.7180	23.9096	13.9808	13.0660	10.4665
	SDRL	31.7368	19.5265	10.4423	9.3629	7.2980
(0.25, 1.25)	ARL	20.1308	14.3351	8.8991	7.9979	6.6808
	SDRL	16.7697	10.9543	5.8205	5.0373	4.1192
(0.5, 0.5)	ARL	13.8956	8.7692	4.9089	4.2280	3.6076
	SDRL	6.1113	3.6136	2.0199	1.6951	1.4404
(0.5, 1.1)	ARL	12.6875	8.5693	4.9286	4.3576	3.6779
	SDRL	9.0614	5.9145	3.0995	2.6394	2.1681
(0.5, 1.25)	ARL	10.7520	7.4277	4.5655	3.9881	3.4040
	SDRL	8.1630	5.1212	2.8383	2.4397	1.9769

Table 7 CDP for different values of n when $\sigma_\varepsilon^2 = 0.5,\ \lambda = 0.1, k = 1$

(δ, ψ)	*n* / *Signal factor*	*3*	*5*	*10*	*12*	*15*
(0.25, 1)	Correct diagnosis percent	89.75	93.25	96.66	97.42	98.13
(0.5, 1)	Correct diagnosis percent	95.00	97.39	99.00	99.18	99.43
(0.75, 1)	Correct diagnosis percent	95.73	97.77	98.96	99.30	99.64
(0, 0.5)	Correct diagnosis percent	99.76	99.90	99.98	99.92	99.97
(0, 1.1)	Correct diagnosis percent	61.34	68.36	78.16	80.57	82.64
(0, 1.25)	Correct diagnosis percent	76.09	82.25	87.50	88.83	90.05

6 Conclusions and future study

In this paper, we investigated the effect of measurement error on detecting and diagnosing performance of MAX-EWMAMS control chart in Phase II. Taking multiple measurements on each sample point as a remedial approach was utilised to compensate for the errors effect. The results of simulation study revealed that the measurement errors affect the performance of MAX-EWMAMS control chart in detecting different step changes. However, using the multiple measurement approach decrease the adverse effect of measurement errors on simultaneous monitoring of process mean and variability. The result also showed that the effect of measurement errors on diagnosing performance of MAX-EWMAMS chart is negligible. As a future study, it is recommended to investigate the effect of gauge measurement errors on the performance of adaptive control charts.

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