Research Article

Quality and Reliability Engineering International

(wileyonlinelibrary.com) DOI: 10.1002/qre.1902

Published online 16 September 2015 in Wiley Online Library

On the Effect of Measurement Error with Linearly Increasing-Type Variance on Simultaneous Monitoring of Process Mean and Variability

Adel Khati Dizabadi,[a] Mahmoud Shahrokhi[a] and Mohammad Reza Maleki[b*†]

In most quality control applications, the errors generated from measurement system can adversely affect the ability of control charts in detecting out-of-control conditions. In this paper, the effect of measurement error with linearly increasing-type variance on the performance of maximum exponentially weighted moving average and mean-squared deviation (MAX-EWMAMS) control chart is studied. For this purpose, different out-of-control scenarios including mean shifts, variance shifts, and simultaneous shifts in both are considered, and the detecting performance of the proposed approach is investigated through simulation study. The results of simulation study in terms of three criteria including average run length, standard deviation of run lengths, and the empirical distribution of run lengths prove that the measurement error with linearly increasing-type variance can adversely affect the performance of MAX-EWMAMS control. Copyright © 2015 John Wiley & Sons, Ltd.

Keywords: measurement error; MAX-EWMAMS control chart; average run length; simultaneous monitoring; linearly increasing-type variance

1. Introduction

The errors in the measurement system are the differences between the actual and the observed values that are known as measurement errors. The measurement errors can be originated from environmental factors (such as temperature, light, and humidity), measuring instruments and operators. Due to the errors of measurement system in most production environments, we are not able to observe the quality characteristics we are interested to monitor. The measurement errors considerably affect the ability of control charts in detecting out-of-control conditions. Consequently, neglecting the effect of measurement error can cause misleading interpretation of control chart signals. The effect of measurement error on the performance of different control charts is well documented in the literature.

In one of the first works in measurement error, Bennett[1] studied the effect of measurement error on monitoring the process mean. He considered the model $Y=X+\varepsilon$, where X is the actual value of the quality characteristic while Y is its observed value. He assumed that both quality characteristics X and Y are normally distributed. He pointed out that in situations where the variance of measurement error is smaller than the variance of the process, it can be overlooked. Then, some other researchers such as Abraham[2] studied the effect of measurement error on the performance of control schemes using the same model in Bennett[1]. Kanazuka[3] also using the same model by Bennett[1] investigated the effect of measurement error on the performance of joint $\overline{X}/R$ control chart. He found that the measurement error decreases the power of control chart in detecting out-of-control conditions. He concluded that using larger sample sizes leads to increasing to the power of the control chart. Mittag[4] considered the same model by Bennett[1] and examined the effect of measurement error on the performance of Shewhart-type $\overline{X}/S$ control chart.

Mittag and Stemann[5] studied the effect of measurement error on the performance of joint $\overline{X}/S$ control chart. They proved that the measurement error can adversely affect the performance of the control chart in detecting out-of-control disturbances. Linna and Woodall[6] investigated the effect of measurement error on the performance of $\overline{X}$ and S control charts using a linear covariate model

[a]Industrial Engineering Department, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran
[b]Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran
*Correspondence to: Mohammad Reza Maleki, Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran.
†E-mail: Reza.maleki@shahed.ac.ir

$Y = A + BX + \varepsilon$. They proved that the variance of the measurement error and the value of parameter B affect the ability of both control charts in detecting a given change in the mean and variance of the process. Linna *et al.*[7] extended the linear covariate model $Y = A + BX + \varepsilon$ to a multivariate case. He showed that the measurement error can considerably deteriorate the detection capability of a multivariate control chart. Stemann and Weihs[8] examined the effect of measurement error on the performance of exponential weighted moving average (EWMA)-based control charts for monitoring both process mean and process variance.

Maravelakis *et al.*[9] investigated the effect of measurement error on the ability of the EWMA control chart in detecting out-of-control conditions in the case of mean shifts. They also examined multiple measurements on each sampled unit to decrease undesirable effect of measurement error. Huwang and Hung[10] studied the effect of measurement error on the performance of two control chart schemes for monitoring process variability in the case of multivariate quality characteristics. Abbasi[11] investigated the performance of the EWMA control chart in the presence of two-component measurement error. He concluded that multiple measurements at each sample point can reduce the effect of measurement error. Costa and Castagliola[12] studied the performance of $\overline{X}$ control chart in the presence of both measurement error and autocorrelation. Scagliarini[13] examined the effect of measurement error on multivariate process capability where the capability indices are computed based on the principal components analysis. Chakraborty and Khurshid[14] studied the effect of measurement error on the power of control chart in situations with zero-truncated Poisson distribution. They pointed out that increasing the sample size can decrease the effect of measurement error on detecting performance of the control chart.

Haq *et al.*[15] studied the effect of measurement error on the performance of EWMA control chart for monitoring process mean based on ranked set sampling (RSS), median RSS (MRSS), imperfect RSS, and imperfect MRSS schemes. They suggested the multiple measurements and non-constant error variance in order to cover undesirable effect of measurement error. Considering the literature, we found that in most researches, the variance of measurement error is assumed to be constant. On the other hand, to the best of our knowledge, there is no research available in the literature that incorporates the measurement error in simultaneous monitoring of process mean and variability. In order to fill this gap, in this paper, we explore the effect of measurement error with linearly increasing-type variance on detection capability of maximum exponentially weighted moving average and mean-squared deviation (MAX-EWMAMS) control chart in detecting mean shifts, variance shifts, and simultaneous shifts in both process mean and variance. Three criteria based on run length values including average run length (ARL), standard deviation of run lengths (SDRLs), and the empirical distribution of run lengths are used in order to assess the MAX-EWMAMS control chart in the presence of measurement error. The run length criterion is defined as the number of consecutive samples taken until the first sample statistic falls outside the control limits interval. In in-control and out-of-control situations, the expected value of run lengths is called in-control ARL (ARL_0) and out-of-control ARL (ARL_1), respectively.

The rest of this paper is organized as follows. In Section 2, we explain a simultaneous monitoring approach called MAX-EWMAMS control chart. In Section 3, we incorporate the measurement error with linearly increasing-type variance into MAX-EWMAMS control chart. In Section 4, we provide a numerical example based on simulation and investigate the effect of measurement error on MAX-EWMAMS control chart. Finally, in Section 5, we conclude the main findings and present a future study.

2. Maximum exponentially weighted moving average and mean-squared deviation control chart

The EWMA-type control charts take into account both the current and previous samples of the process. Hence, they are more sensitive to small shifts rather than Shewhart-type control charts. In this section, we describe a simultaneous monitoring control scheme called MAX-EWMAMS control chart proposed by Memar and Niaki[16] based on two control charts including EWMA and exponential weighted mean square error (EWMS) control charts.

2.1. Exponential weighted moving average control chart

Supposed that when the process is in-control, the true value of quality characteristic X under investigation follows a normal distribution with parameters μ_0 and σ_0^2. Let $\overline{X}_t$ be the sample mean that is taken at time $t; t = 1, 2, \ldots$ based on random samples of size n. The EWMA control statistic for monitoring the process mean corresponding to sample $t, t = 1, 2, \ldots$ is defined according to Equation (1):

$$Z_t = \lambda \overline{X}_t + (1 - \lambda) Z_{t-1}, \tag{1}$$

where $\lambda; 0 < \lambda \leq 1$ is the smoothing parameter and $Z_0 = \mu_0$.

2.2. Exponential weighted mean square error control chart

The EWMS control chart statistic for monitoring the process variability that is first proposed by MacGregor and Harris[17] plots against the sample sequence $t; t = 1, 2, \ldots$ as follows:

$$S_t^2 = (1 - \lambda) S_{t-1}^2 + \lambda \sum_{k=1}^{n} \frac{(X_{tk} - \mu_0)^2}{n}, \tag{2}$$

where X_{tk} is the kth observation in the tth sample and $S_0^2 = \sigma_0^2$. It can be statistically checked that

$$E\left[S_t^2\right] = \sigma_0^2, \tag{3}$$

$$Var\left[S_t^2\right] = \frac{2\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma_0^4. \tag{4}$$

It can be shown that in situations where the observations are independent and normally distributed, for each value of sample size n, when $t \to \infty$, then we have the following:

Table I. Distribution of run lengths in detecting mean shifts for different values of D when $C = 0$

h_m		3.0799	3.1137	3.2110	3.2451	3.2509
δ		0	1	2	3	5
0	ARL	367.0275	371.6275	373.5350	374.3840	367.0725
	SDRL	354.1366	385.1625	490.4916	531.1813	543.8256
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.18%	43.36%	46.50%	47.48%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.70%	22.35%	21.33%	19.53%	18.22%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.72%	15.43%	13.53%	12.46%	12.09%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.67%	22.04%	21.78%	21.51%	22.21%
0.25	ARL	163.2075	334.0785	338.6300	341.1155	358.8690
	SDRL	162.1143	366.0217	458.6357	526.4423	542.5478
	$p(RL < ARL - 0.5 \times SDRL)$	38.72%	39.64%	44.26%	44.60%	46.14%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	25.19%	22.91%	20.96%	21.67%	19.57%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.29%	14.71%	13.01%	12.10%	13.22%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.80%	22.74%	21.77%	21.63%	21.08%
0.5	ARL	49.9570	247.0270	287.6900	326.1140	331.7480
	SDRL	45.5500	248.6189	383.1722	432.3062	465.8325
	$p(RL < ARL - 0.5 \times SDRL)$	39.23%	39.40%	42.56%	45.66%	46.38%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.95%	24.33%	23.43%	20.84%	20.79%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	13.73%	13.66%	13.83%	12.61%	11.56%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.09%	22.61%	20.18%	20.89%	21.27%
1	ARL	11.1115	107.6130	166.6245	208.1495	249.5790
	SDRL	8.0708	112.7888	213.4826	301.5726	366.2184
	$p(RL < ARL - 0.5 \times SDRL)$	39.14%	39.26%	43.62%	44.36%	46.17%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.97%	23.28%	21.63%	21.76%	20.78%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	13.78%	14.50%	13.45%	12.49%	12.91%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.11%	22.96%	21.30%	21.39%	20.14%
1.5	ARL	4.9400	50.6605	94.3915	124.8615	162.0705
	SDRL	2.8646	49.3697	116.5436	166.5957	252.6518
	$p(RL < ARL - 0.5 \times SDRL)$	34.05%	38.43%	41.93%	45.06%	44.92%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	30.31%	24.53%	23.10%	19.43%	21.10%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	9.09%	14.77%	14.13%	14.36%	13.14%
	$p(RL \ge ARL + 0.5 \times SDRL)$	26.55%	22.27%	20.84%	21.15%	20.84%
2	ARL	3.0795	27.0255	49.5900	73.4730	110.3555
	SDRL	1.5608	24.1124	64.9632	103.0322	169.8044
	$p(RL < ARL - 0.5 \times SDRL)$	41.30%	38.66%	42.96%	44.30%	45.75%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.46%	23.23%	21.33%	21.00%	20.87%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	0%	15.33%	13.07%	12.49%	12.88%
	$p(RL \ge ARL + 0.5 \times SDRL)$	34.24%	22.78%	22.64%	22.21%	20.50%
2.5	ARL	2.1870	16.4085	32.0085	47.4725	79.0220
	SDRL	1.0312	13.7585	34.6826	60.9977	106.1840
	$p(RL < ARL - 0.5 \times SDRL)$	28.61%	39.00%	41.32%	42.71%	44.36%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	40.87%	24.07%	22.06%	21.56%	21.74%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	0%	12.90%	14.42%	13.70%	12.32%
	$p(RL \ge ARL + 0.5 \times SDRL)$	30.52%	24.03%	22.20%	22.03%	21.58%
3	ARL	1.6645	11.3660	21.2825	31.8325	54.1715
	SDRL	0.7154	8.4861	23.0186	41.1190	74.1377
	$p(RL < ARL - 0.5 \times SDRL)$	47.57%	34.20%	39.66%	41.95%	43.96%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	0%	28.31%	22.73%	22.93%	20.92%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	40.80%	15.32%	15.26%	12.44%	13.45%
	$p(RL \ge ARL + 0.5 \times SDRL)$	11.63%	22.17%	22.35%	22.68%	21.67%

ARL, average run length; SDRL, standard deviation of run length.

$$S_t^2/\sigma_0^2 \to \chi_v^2(v = n(2-\lambda)/\lambda. \quad (5)$$

2.3. Maximum exponentially weighted moving average and mean-squared deviation control chart

In order to derive MAX-EWMAMS statistic, Memar and Niaki[16] used EWMA and EWMS statistics for monitoring the process mean and process variability, respectively. Then, they transformed the distribution of both statistics to a similar distribution (standard normal

Table II. Distribution of run lengths in detecting mean shifts for different values of C when $D=1$

h_m		3.0799	3.1137	3.1509	3.1686	3.1793
δ		No error	0	1	2	3
0	ARL	367.0275	371.6275	371.5484	374.4776	371.7916
	SDRL	354.1366	385.1625	435.5955	449.6847	462.4236
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.18%	40.65%	41.46%	41..89%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.70%	22.35%	22.76%	22.56%	22.37%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.72%	15.43%	14.29%	14.38%	12.90%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.67%	22.04%	22.30%	21.60%	22.84%
0.25	ARL	163.2075	334.0785	339.6170	346.3175	347.3855
	SDRL	162.1143	366.0217	379.3418	389.8599	434.0846
	$p(RL < ARL - 0.5 \times SDRL)$	38.72%	39.64%	41.20%	40.50%	40.28%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	25.19%	22.91%	22.58%	23.56%	25.29%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.29%	14.71%	13.20%	15.28%	14.43%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.80%	22.74%	23.02%	20.66%	20.00%
0.5	ARL	49.9570	247.0270	258.9660	266.6970	266.9620
	SDRL	45.5500	248.6189	294.8828	321.6827	340.1355
	$p(RL < ARL - 0.5 \times SDRL)$	39.23%	39.40%	40.53%	40.95%	40.57%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.95%	24.33%	22.98%	23.51%	23.89%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	13.73%	13.66%	13.96%	13.53%	13.91%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.09%	22.61%	22.53%	22.01%	21.63%
1	ARL	11.1115	107.6130	120.9345	124.0437	143.7665
	SDRL	8.0708	112.7888	133.5202	157.7225	159.6814
	$p(RL < ARL - 0.5 \times SDRL)$	39.14%	39.26%	40.26%	41.35%	41.57%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.97%	23.28%	24.35%	22.26%	22.33%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	13.78%	14.50%	12.90%	14.54%	14.64%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.11%	22.96%	22.49%	21.85%	21.46%
1.5	ARL	4.9400	50.6605	57.4940	59.7770	64.7955
	SDRL	2.8646	49.3697	57.7035	67.6889	72.0818
	$p(RL < ARL - 0.5 \times SDRL)$	34.05%	38.43%	39.28%	40.22%	41.59%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	30.31%	24.53%	23.69%	23.32%	21.96%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	9.09%	14.77%	15.20%	13.96%	14.44%
	$p(RL \ge ARL + 0.5 \times SDRL)$	26.55%	22.27%	21.83%	22.50%	22.01%
2	ARL	3.0795	27.0255	29.4990	33.1303	36.9290
	SDRL	1.5608	24.1124	30.9345	32.4019	38.3711
	$p(RL < ARL - 0.5 \times SDRL)$	41.30%	38.66%	38.31%	38.77%	39.55%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.46%	23.23%	23.90%	23.60%	24.09%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	0%	15.33%	14.76%	15.61%	13.20%
	$p(RL \ge ARL + 0.5 \times SDRL)$	34.24%	22.78%	23.03%	22.02%	23.16%
2.5	ARL	2.1870	16.4085	18.5545	19.9887	22.1680
	SDRL	1.0312	13.7585	16.0871	19.3639	21.3746
	$p(RL < ARL - 0.5 \times SDRL)$	28.61%	39.00%	35.57%	38.39%	38.51%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	40.87%	24.07%	27.96%	23.83%	24.03%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	0%	12.90%	14.09%	13.46%	14.00%
	$p(RL \ge ARL + 0.5 \times SDRL)$	30.52%	24.03%	22.38%	24.32%	23.46%
3	ARL	1.6645	11.3660	12.2885	13.2520	14.5440
	SDRL	0.7154	8.4861	10.2163	11.3627	13.3611
	$p(RL < ARL - 0.5 \times SDRL)$	47.57%	34.20%	33.75%	37.82%	35.92%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	0%	28.31%	29.57%	25.53%	26.48%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	40.80%	15.32%	14.40%	14.73%	15.03%
	$p(RL \ge ARL + 0.5 \times SDRL)$	11.63%	22.17%	22.28%	21.92%	22.57%

ARL, average run length; SDRL, standard deviation of run length.

distribution). After that, due to the same distribution of both transformed statistics, they used a single control chart for monitoring the simultaneous process mean and process variability. It can be statistically checked that the distribution of Z_t in Equation (1) is as follows:

$$N\left(\mu_0, \frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\sigma_0^2\right]\right. \quad (6)$$

Obviously, U_t that is used for monitoring the process mean follows the standard normal distribution as follows:

$$U_t = \frac{(Z_t - \mu_0)}{\sqrt{\frac{\lambda}{n(2-\lambda)}\left[1-(1-\lambda)^{2t}\sigma_0^2\right]}}. \quad (7)$$

Table III. Distribution of run lengths in detecting variance shifts for different values of D when $C = 0$

h_m		3.0799	3.1137	3.2110	3.2451	3.2509
ψ		0	1	2	3	5
1	ARL	367.0275	371.6275	373.5350	374.3840	367.0725
	SDRL	354.1366	385.1625	490.4916	531.1813	543.8256
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.18%	43.36%	46.50%	47.48%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.70%	22.35%	21.33%	19.53%	18.22%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.72%	15.43%	13.53%	12.46%	12.09%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.67%	22.04%	21.78%	21.51%	22.21%
1.1	ARL	137.6755	315.0215	330.0015	351.3370	362.1805
	SDRL	141.5861	350.6968	473.9762	511.2236	531.8964
	$p(RL < ARL - 0.5 \times SDRL)$	39.58%	37.65%	44.17%	45.60%	46.71%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	22.93%	26.30%	21.84%	21.17%	19.68%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.90%	14.32%	12.39%	11.69%	12.66%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.59%	21.73%	21.60%	21.54%	20.95%
1.25	ARL	49.2310	258.0715	294.6655	339.7250	335.6975
	SDRL	46.9689	277.8653	419.4686	461.1941	483.1959
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.13%	43.78%	45.52%	46.87%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.09%	24.48%	21.01%	20.91%	19.60%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.53%	13.22%	12.61%	12.52%	11.78%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.47%	22.17%	22.60%	21.05%	21.75%
1.5	ARL	17.6835	178.8330	256.5505	277.3980	303.7830
	SDRL	16.0772	191.7247	343.5926	398.5305	481.9449
	$p(RL < ARL - 0.5 \times SDRL)$	38.65%	38.26%	42.15%	44.30%	47.30%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	23.26%	24.65%	22.30%	20.75%	19.08%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	16.65%	15.00%	13.69%	13.95%	12.02%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.44%	22.09%	21.86%	21.00%	21.60%
1.75	ARL	9.3385	130.3210	191.0090	253.3105	271.9225
	SDRL	8.2469	130.5434	273.2633	345.2322	416.4403
	$p(RL < ARL - 0.5 \times SDRL)$	39.95%	38.30%	43.73%	45.56%	46.14%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	23.30%	25.26%	22.30%	20.31%	20.71%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.14%	14.82%	12.19%	11.86%	12.75%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.61%	21.62%	21.78%	22.27%	20.40%
2	ARL	6.1245	92.4865	148.9000	214.1720	245.5625
	SDRL	5.1539	96.7206	201.9542	280.2457	360.2203
	$p(RL < ARL - 0.5 \times SDRL)$	38.13%	38.73%	43.60%	44.69%	44.60%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.18%	25.63%	20.34%	21.58%	22.21%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.39%	13.73%	15.00%	12.44%	12.33%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.30%	21.91%	21.06%	21.79%	20.86%
2.5	ARL	3.9010	47.8060	97.1500	146.4330	183.3755
	SDRL	3.1047	49.0537	119.9116	177.3190	264.3480
	$p(RL < ARL - 0.5 \times SDRL)$	40.95%	39.26%	42.40%	44.97%	45.08%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.08%	24.58%	22.08%	20.04%	20.09%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	8.21%	14.82%	13.39%	13.34%	12.82%
	$p(RL \ge ARL + 0.5 \times SDRL)$	24.76%	21.34%	22.13%	21.65%	22.01%

ARL, average run length; SDRL, standard deviation of run length.

Memar and Niaki[16] also used the following statistic with approximate standard normal distribution for monitoring the process variability according to Equation (8):

$$V_t = \phi^{-1}\left[H\left\{\frac{vS_t^2}{\sigma_0^2}; v\right\}\right], \tag{8}$$

where $H\{a; d\}$ is defined as the cumulative distribution function of a chi-square distribution denoted by $H\{a; d\} = pr\{\chi_d^2 \le a\}$. The MAX-EWMAMS statistic at sample point $t; t = 1, 2, \ldots$ is defined as follows:

$$M_t = \max\{|U_t|, |V_t|\}. \tag{9}$$

Because $M_t \ge 0$, the MAX-EWMAMS control chart only has upper control limit.

Table IV. Distribution of run lengths in detecting variance shifts for different values of C when $D = 1$

ψ		No error	0	1	2	3
1	ARL	367.0275	371.6275	371.5484	374.4776	371.7916
	SDRL	354.1366	385.1625	435.5955	449.6847	462.4236
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.18%	40.65%	41.46%	41.89%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.70%	22.35%	22.76%	22.56%	22.37%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.72%	15.43%	14.29%	14.38%	12.90%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.67%	22.04%	22.30%	21.60%	22.84%
1.1	ARL	137.6755	315.0215	335.7790	342.9350	345.3760
	SDRL	141.5861	350.6968	376.6129	397.5227	408.3486
	$p(RL < ARL - 0.5 \times SDRL)$	39.58%	37.65%	40.91%	40.82%	42.38%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	22.93%	26.30%	21.80%	23.40%	21.33%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.90%	14.32%	14.82%	14.34%	13.62%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.59%	21.73%	22.47%	21.44%	22.67%
1.25	ARL	49.2310	258.0715	273.9095	283.5640	288.0515
	SDRL	46.9689	277.8653	311.8862	354.4435	364.4668
	$p(RL < ARL - 0.5 \times SDRL)$	38.91%	40.13%	40.47%	40.82%	40.33%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	24.09%	24.48%	22.95%	23.45%	23.95%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.53%	13.22%	13.92%	13.52%	14.30%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.47%	22.17%	22.66%	22.21%	21.42%
1.5	ARL	17.6835	178.8330	193.4594	205.5740	223.6100
	SDRL	16.0772	191.7247	228.3256	244.5805	276.3315
	$p(RL < ARL - 0.5 \times SDRL)$	38.65%	38.26%	39.83%	40.34%	41.09%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	23.26%	24.65%	22.91%	24.13%	22.04%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	16.65%	15.00%	15.22%	14.58%	14.66%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.44%	22.09%	22.04%	20.95%	22.21%
1.75	ARL	9.3385	130.3210	141.0470	150.0370	162.3905
	SDRL	8.2469	130.5434	158.2259	175.4404	183.7114
	$p(RL < ARL - 0.5 \times SDRL)$	39.95%	38.30%	40.00%	41.82%	41.71%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	23.30%	25.26%	23.95%	21.60%	22.45%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.14%	14.82%	14.31%	14.80%	13.23%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.61%	21.62%	21.74%	21.78%	22.61%
2	ARL	6.1245	92.4865	96.3040	104.7500	113.5995
	SDRL	5.1539	96.7206	111.1757	128.8469	140.4171
	$p(RL < ARL - 0.5 \times SDRL)$	38.13%	38.73%	38.82%	40.47%	41.38%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.18%	25.63%	24.26%	22.79%	22.71%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.39%	13.73%	15.39%	14.01%	14.15%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.30%	21.91%	21.53%	22.73%	21.76%
2.5	ARL	3.9010	47.8060	51.7385	55.3060	64.1345
	SDRL	3.1047	49.0537	55.9597	65.2728	75.6675
	$p(RL < ARL - 0.5 \times SDRL)$	40.95%	39.26%	39.56%	40.49%	40.95%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.08%	24.58%	24.19%	22.73%	23.52%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	8.21%	14.82%	14.17%	14.13%	14.57%
	$p(RL \ge ARL + 0.5 \times SDRL)$	24.76%	21.34%	22.08%	22.65%	20.96%

ARL, average run length; SDRL, standard deviation of run length.

3. Incorporating measurement error with linearly increasing-type variance into MAX-EWMAMS control chart

As noted, due to the measurement error, we are not able to observe the true value of quality characteristic X under investigation. Instead of X, we can observe and monitor Y that is related to quality characteristic X according to the following equation:

$$Y = A + BX + \varepsilon, \tag{10}$$

where A and B are intercept and slope constants and ε is the random error that is independent of X and follows a normal distribution with mean zero. Recall that in most researches, the variance of the error term is considered as a constant value. However, in some production systems, this assumption is violated. For example, in some practical environments, the variance of measurement error depends on the mean level of the process (Montgomery and Runger[18] and Linna and Woodall[6]). In this paper, we assume that the variance of measurement error term changes linearly with quality characteristic under investigation, that is, $C + D\mu_0$. Consequently, Y follows a normal distribution as follows:

$$N\left(A + B\mu_0, B^2\sigma_0^2 + C + D\mu_0\right). \tag{11}$$

Table V. Distribution of run lengths in detecting simultaneous shifts for different values of D when $C = 0$ and $\delta = 0.5$

h_m		3.0799	3.1137	3.2110	3.2451	3.2509
ψ		0	1	2	3	5
1.1	ARL	36.1595	216.2660	274.0185	298.8975	318.4225
	SDRL	32.7293	229.0723	351.1964	441.4029	467.8540
	$p(RL < ARL - 0.5 \times SDRL)$	39.54%	39.36%	44.44%	45.64%	46.75%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	22.36%	23.86%	19.65%	20.53%	19.06%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	15.23%	14.27%	12.36%	12.18%	11.75%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.87%	22.51%	23.55%	21.65%	22.44%
1.25	ARL	22.6325	184.6195	243.4985	284.7960	298.8835
	SDRL	20.4230	203.2850	345.6469	419.8624	462.0676
	$p(RL < ARL - 0.5 \times SDRL)$	36.86%	39.31%	42.88%	44.75%	47.02%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.05%	24.04%	22.67%	22.26%	19.83%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.73%	13.95%	13.43%	12.40%	11.74%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.36%	22.70%	21.02%	20.59%	21.41%
1.5	ARL	12.4925	139.7600	200.8180	256.4210	277.4820
	SDRL	10.7629	139.6988	255.7863	344.0812	393.9189
	$p(RL < ARL - 0.5 \times SDRL)$	36.86%	39.23%	42.57%	44.22%	45.24%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	25.79%	24.36%	21.37%	21.20%	21.57%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	14.95%	13.51%	13.86%	13.73%	12.93%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.40%	22.90%	22.20%	20.85%	20.26%
1.75	ARL	8.0350	102.3600	162.0800	217.0740	248.2665
	SDRL	6.9166	106.2542	216.0504	300.1526	358.3649
	$p(RL < ARL - 0.5 \times SDRL)$	38.59%	39.77%	42.17%	43.95%	46.40%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	26.45%	22.50%	21.91%	22.00%	20.40%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	12.26%	15.95%	15.02%	12.72%	12.35%
	$p(RL \ge ARL + 0.5 \times SDRL)$	22.77%	21.78%	20.90%	21.33%	20.85%
2	ARL	5.6635	75.0010	134.6650	180.2250	228.3665
	SDRL	4.7517	74.7526	171.6041	256.1712	328.3209
	$p(RL < ARL - 0.5 \times SDRL)$	40.31%	38.97%	41.99%	45.24%	47.20%
	$p(ARL - 0.5 \times SDRL \le RL < ARL)$	19.63%	24.62%	23.67%	20.22%	19.37%
	$p(ARL \le RL < ARL + 0.5 \times SDRL)$	19.02%	14.10%	12.74%	13.65%	11.28%
	$p(RL \ge ARL + 0.5 \times SDRL)$	21.04%	22.31%	21.60%	20.89%	22.15%
2.5	ARL	3.8330	41.5255	83.3175	124.7760	172.1530
	SDRL	3.0389	43.6927	108.9257	166.2782	251.2772
	$p(rl < ARL - 0.5 \times SDRL)$	42.86%	39.15%	42.72%	45.07%	46.44%
	$p(ARL - 0.5 \times SDRL \le rl < ARL)$	14.73%	23.37%	21.77%	21.46%	19.51%
	$p(ARL \le rl < ARL + 0.5 \times SDRL)$	20.22%	15.22%	13.91%	12.01%	12.80%
	$p(rl \ge ARL + 0.5 \times SDRL)$	22.19%	22.26%	21.60%	21.46%	21.25%

ARL, average run length; SDRL, standard deviation of run length.

Incorporating the measurement error with linearly increasing variance, we rewrite the EWMA statistic for sample $t; t = 1, 2, \dots$ in Equation (1) as follows:

$$Z_t = \lambda \overline{Y}_t + (1-\lambda) Z_{t-1}, \tag{12}$$

where $\overline{Y}_t$ *is the mean of observed* Ys over the tth sample size of n and $Z_0 = A + B\mu_0$. It can be statistically proved that Z_t follows a normal distribution with the following parameters:

$$N\left(A + B\mu_0, \left(\frac{\lambda}{n(2-\lambda)}\right)\left[1-(1-\lambda)^{2t}\right] \times \left(B^2\sigma_0^2 + C + D\mu_0\right)\right). \tag{13}$$

Consequently, the extended U_t statistic for monitoring the process mean in the presence of measurement error with linearly increasing-type variance follows a standard normal distribution as follows:

$$U_t' = \frac{Z_t - (A + B\mu_0)}{\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right] \times \frac{B^2\sigma_0^2 + C + D\mu_0}{n}}}. \tag{14}$$

Table VI. Distribution of run lengths in detecting simultaneous shifts for different values of C when $D = 1$ and $\delta = 0.5$

h_m		3.0799	3.1137	3.1509	3.1686	3.1793
ψ		No error	0	1	2	3
1.1	ARL	36.1595	216.2660	228.5460	245.0075	251.5956
	SDRL	32.7293	229.0723	271.7987	288.5407	305.1125
	$p(RL < ARL - 0.5 \times SDRL)$	39.54%	39.36%	39.30%	40.41%	42.13%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	22.36%	23.86%	25.06%	23.64%	22.62%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	15.23%	14.27%	14.31%	15.42%	14.08%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.87%	22.51%	21.33%	20.53%	21.17%
1.25	ARL	22.6325	184.6195	196.9090	207.9535	210.2105
	SDRL	20.4230	203.2850	226.3482	251.7451	269.1652
	$p(RL < ARL - 0.5 \times SDRL)$	36.86%	39.31%	38.98%	41.46%	40.88%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	26.05%	24.04%	25.06%	21.82%	23.93%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	14.73%	13.95%	13.65%	14.35%	13.37%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.36%	22.70%	22.31%	22.37%	21.82%
1.5	ARL	12.4925	139.7600	151.0165	159.3100	172.2805
	SDRL	10.7629	139.6988	168.4158	187.6774	197.1766
	$p(RL < ARL - 0.5 \times SDRL)$	36.86%	39.23%	39.86%	41.77%	41.42%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	25.79%	24.36%	23.11%	21.38%	22.46%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	14.95%	13.51%	14.35%	15.24%	13.95%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.40%	22.90%	22.68%	21.61%	22.17%
1.75	ARL	8.0350	102.3600	116.6915	120.7680	131.6855
	SDRL	6.9166	106.2542	128.1981	140.1855	161.8334
	$p(RL < ARL - 0.5 \times SDRL)$	38.59%	39.77%	39.73%	40.00%	40.04%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	26.45%	22.50%	23.42%	23.95%	25.15%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	12.26%	15.95%	14.84%	13.03%	14.24%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.77%	21.78%	22.01%	23.02%	20.57%
2	ARL	5.6635	75.0010	84.4425	89.3390	96.7955
	SDRL	4.7517	74.7526	91.4282	104.8821	114.0955
	$p(RL < ARL - 0.5 \times SDRL)$	40.31%	38.97%	39.15%	40.88%	41.24%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	19.63%	24.62%	24.04%	23.15%	22.48%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	19.02%	14.10%	14.90%	13.46%	13.33%
	$p(RL \geq ARL + 0.5 \times SDRL)$	21.04%	22.31%	21.91%	22.51%	22.95%
2.5	ARL	3.8330	41.5255	45.5920	53.0815	56.1230
	SDRL	3.0389	43.6927	51.5382	58.3925	70.5022
	$p(RL < ARL - 0.5 \times SDRL)$	42.86%	39.15%	39.61%	40.22%	39.91%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	14.73%	23.37%	24.00%	24.17%	24.40%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	20.22%	15.22%	14.84%	13.91%	14.85%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.19%	22.26%	21.55%	21.70%	20.84%

ARL, average run length; SDRL, standard deviation of run length.

The extended V_t statistic for monitoring the process variability in the presence of measurement error with linearly increasing-type variance with approximate standard normal distribution can be rewritten as follows:

$$V_t^{'} = \phi^{-1}\left[H\left\{\frac{vS_t^2}{\sigma_y^2}; v\right\}\right], \tag{15}$$

where $\sigma_y^2 = B^2\sigma_0^2 + C + D\mu_0$, $v = \frac{n(2-\lambda)}{\lambda}$, *and* $H\{a; d\} = pr\{\chi_d^2 \leq a\}$ *are the cumulative distribution function of a chi-square distribution. The MAX-EWMAMS statistic for simultaneous monitoring of the process mean and variability in the presence of measurement error with linearly increasing variance at sample point* $t; t = 1, 2, \ldots$ is defined as follows:

$$M_t^{'} = \max\{|U_t^{'}|, |V_t^{'}|\}. \tag{16}$$

It is obvious in Equation (16) that large values of $M_t^{'}$ can cause out-of-control signals. Statistically approximating the distribution of $M_t^{'}$ is not easily possible. In this paper, the upper control limit of the proposed control chart is estimated through the simulation experiments so that the in-control ARL (ARL_0) be equal to a predetermined value.

Table VII. Distribution of run lengths in detecting simultaneous shifts for different values of D when $C=0$ and $\delta=1$

h_m		3.0799	3.1137	3.2110	3.2451	3.2509
ψ		0	1	2	3	5
1.1	ARL	10.3545	105.2530	160.3300	213.0145	241.1145
	SDRL	7.9440	106.8790	215.9459	300.1392	365.3734
	$p(RL < ARL - 0.5 \times SDRL)$	38.32%	39.66%	42.57%	44.77%	46.13%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	24.38%	23.74%	22.45%	20.71%	19.82%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	12.48%	13.95%	13.75%	13.66%	12.30%
	$p(RL \geq ARL + 0.5 \times SDRL)$	24.82%	22.65%	21.23%	20.86%	21.75%
1.25	ARL	8.7675	87.2175	146.3890	180.8515	230.5500
	SDRL	7.0110	91.6396	196.9995	261.6435	342.0095
	$p(RL < ARL - 0.5 \times SDRL)$	38.16%	39.77%	43.19%	44.72%	45.82%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	20.99%	24.05%	21.82%	21.02%	20.39%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	18.77%	14.12%	13.01%	13.17%	12.95%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.08%	22.06%	21.98%	21.09%	20.84%
1.5	ARL	6.9320	77.6950	129.8170	171.3990	209.2105
	SDRL	5.2047	72.9625	162.3571	236.7214	302.9830
	$p(RL < ARL - 0.5 \times SDRL)$	40.84%	39.54%	43.06%	44.43%	46.55%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	23.19%	23.57%	22.56%	21.35%	19.42%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	10.37%	14.88%	12.92%	12.59%	12.67%
	$p(RL \geq ARL + 0.5 \times SDRL)$	25.60%	22.01%	21.46%	21.63%	21.36%
1.75	ARL	5.7970	59.4345	106.6280	151.9065	195.2725
	SDRL	4.6852	62.0857	147.8613	210.4807	299.6907
	$p(RL < ARL - 0.5 \times SDRL)$	39.29%	38.84%	42.90%	44.48%	46.23%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	21.11%	24.34%	22.42%	21.03%	20.03%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	19.21%	14.49%	12.76%	13.31%	12.53%
	$p(RL \geq ARL + 0.5 \times SDRL)$	20.39%	22.33%	21.92%	21.18%	21.21%
2	ARL	4.6315	49.0485	88.8490	130.8295	175.5850
	SDRL	3.6144	48.4471	119.6174	169.1622	266.9163
	$p(rl < ARL - 0.5 \times SDRL)$	34.03%	38.58%	42.79%	44.28%	46.15%
	$p(ARL - 0.5 \times SDRL \leq rl < ARL)$	26.35%	24.26%	21.97%	21.10%	19.66%
	$p(ARL \leq rl < ARL + 0.5 \times SDRL)$	16.18%	14.99%	13.33%	13.19%	12.36%
	$p(rl \geq ARL + 0.5 \times SDRL)$	23.44%	22.17%	21.91%	21.43%	21.83%
2.5	ARL	3.4635	31.9420	64.6910	93.8795	143.9715
	SDRL	2.6931	30.8603	77.8544	131.2938	203.6150
	$p(RL < ARL - 0.5 \times SDRL)$	46.86%	39.49%	42.75%	43.88%	45.12%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	16.38%	23.62%	21.82%	21.34%	20.82%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	11.64%	14.41%	13.28%	13.12%	12.83%
	$p(RL \geq ARL + 0.5 \times SDRL)$	25.12%	22.48%	22.15%	21.66%	21.23%

ARL, average run length; SDRL, standard deviation of run length.

4. Performance evaluation

In this section, the performance of the MAX-EWMAMS control chart in the presence of measurement error with linearly increasing-type variance is investigated through a detailed numerical example based on simulation. All simulation experiments in this section are conducted in MATLAB computer software (MathWorks, Natick, MA, USA). It has been assumed that when the process is in-control, X follows a normal distribution with parameters $\mu_0 = 8$ and $\sigma_0^2 = 1$. Here, we select $n = 1, A = 0, B = 1,$ and $\lambda = 0.25$. For each variance value of error term, we set the control limit of the MAX-EWMAMS control chart so that the ARL_0 value is matched roughly equal to 370. Then, in each column (for each variance value of error term) based on 10,000 replicates, we simulate the performance of the proposed control chart in detecting different step shifts under different out-of-control scenarios. Note that in out-of-control scenarios, the step shifts $\mu_1 = \mu_0 + \delta\sigma_0$ and $\sigma_1 = \psi\sigma_0$ are considered for mean and variance shifts, respectively. The performance of MAX-EWMAMS control chart in the presence of measurement error in detecting mean and variance shifts is displayed in Tables 1, 2 and Tables 3, 4, respectively. The results of detecting simultaneous shifts in both process mean and variability are also summarized in Tables 5–10. Recall that in all simulation experiments, three criteria including ARL, SDRLs, and empirical distribution of run lengths are calculated for different covariate model parameters.

4.1. Investigating mean shifts

In Table I, the results of the proposed MAX-EWMAMS control chart in detecting different mean shifts when $D = 0, 1, 2, 3, 4, 5$ and $C = 0$ are summarized. The results of Table I show that even small values of parameter D can seriously decrease the ability of MAX-EWMAMS

Table VIII. Distribution of run lengths in detecting simultaneous shifts for different values of C when $D = 1$ and $\delta = 1$

h_m		3.0799	3.1137	3.1509	3.1686	3.1793
ψ		No error	0	1	2	3
1.1	ARL	10.3545	105.2530	111.4420	124.0515	126.6040
	SDRL	7.9440	106.8790	128.9749	141.9220	157.8290
	$p(RL < ARL - 0.5 \times SDRL)$	38.32%	39.66%	39.76%	40.30%	41.29%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	24.38%	23.74%	24.46%	23.21%	22.75%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	12.48%	13.95%	13.66%	14.55%	13.70%
	$p(RL \geq ARL + 0.5 \times SDRL)$	24.82%	22.65%	22.12%	21.94%	22.26%
1.25	ARL	8.7675	87.2175	100.3315	110.0360	112.9360
	SDRL	7.0110	91.6396	115.9057	130.8012	135.3714
	$p(RL < ARL - 0.5 \times SDRL)$	38.16%	39.77%	40.33%	40.29%	41.48%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	20.99%	24.05%	23.04%	23.34%	22.65%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	18.77%	14.12%	14.40%	14.70%	13.67%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.08%	22.06%	22.23%	21.67%	22.20%
1.5	ARL	6.9320	77.6950	82.9080	87.8540	98.9620
	SDRL	5.2047	72.9625	88.1170	105.5142	123.3881
	$p(RL < ARL - 0.5 \times SDRL)$	40.84%	39.54%	39.66%	40.56%	40.74%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	23.19%	23.57%	23.84%	23.35%	23.82%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	10.37%	14.88%	14.18%	13.91%	13.47%
	$p(RL \geq ARL + 0.5 \times SDRL)$	25.60%	22.01%	22.32%	22.18%	21.97%
1.75	ARL	5.7970	59.4345	66.6305	76.7735	82.3265
	SDRL	4.6852	62.0857	75.1415	87.8164	94.6538
	$p(RL < ARL - 0.5 \times SDRL)$	39.29%	38.84%	38.87%	40.19%	41.10%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	21.11%	24.34%	24.66%	23.74%	22.54%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	19.21%	14.49%	14.46%	14.18%	13.87%
	$p(RL \geq ARL + 0.5 \times SDRL)$	20.39%	22.33%	22.01%	21.89%	22.49%
2	ARL	4.6315	49.0485	52.5620	60.9250	63.7335
	SDRL	3.6144	48.4471	57.1972	66.7716	75.7590
	$p(RL < ARL - 0.5 \times SDRL)$	34.03%	38.58%	38.84%	39.86%	41.20%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	26.35%	24.26%	25.17%	23.71%	22.07%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	16.18%	14.99%	14.14%	14.34%	14.53%
	$p(RL \geq ARL + 0.5 \times SDRL)$	23.44%	22.17%	21.85%	22.09%	22.20%
2.5	ARL	3.4635	31.9420	34.3680	41.0175	41.9530
	SDRL	2.6931	30.8603	36.5830	46.1803	48.5160
	$p(RL < ARL - 0.5 \times SDRL)$	46.86%	39.49%	39.18%	39.69%	40.15%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	16.38%	23.62%	24.63%	23.72%	22.94%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	11.64%	14.41%	14.14%	14.21%	14.85%
	$p(RL \geq ARL + 0.5 \times SDRL)$	25.12%	22.48%	22.05%	22.38%	22.06%

ARL, average run length; SDRL, standard deviation of run length.

Table IX. Distribution of run lengths in detecting simultaneous shifts for different values of D when $C=0$ and $\delta=2$

h_m		3.0799	3.1137	3.2110	3.2451	3.2509
ψ		0	1	2	3	5
1.1	ARL	3.0805	26.0945	47.7130	77.2270	113.0885
	SDRL	1.6964	23.4642	62.5016	102.2316	162.4089
	$p(RL < ARL - 0.5 \times SDRL)$	43.86%	39.35%	42.00%	43.88%	45.87%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	24.08%	23.68%	23.05%	20.93%	20.30%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	0%	14.37%	13.17%	14.12%	12.26%
	$p(RL \geq ARL + 0.5 \times SDRL)$	32.06%	22.60%	21.78%	21.07%	21.57%
1.25	ARL	3.0660	25.0530	45.9900	73.1585	109.1200
	SDRL	1.7913	23.1117	59.2542	96.8187	158.4487
	$p(RL < ARL - 0.5 \times SDRL)$	47.74%	38.27%	41.79%	43.84%	45.97%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	0%	25.23%	22.54%	21.12%	19.65%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	20.61%	13.77%	13.40%	13.41%	12.83%
	$p(RL \geq ARL + 0.5 \times SDRL)$	31.65%	22.73%	22.27%	21.63%	21.55%
1.5	ARL	2.9855	22.6280	44.1895	65.0525	102.9785
	SDRL	1.9268	20.9672	54.3381	90.6307	147.7314
	$p(RL < ARL - 0.5 \times SDRL)$	49.47%	37.92%	41.55%	42.70%	45.70%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	0%	25.37%	22.13%	22.57%	19.99%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	19.64%	14.09%	14.14%	13.80%	12.93%
	$p(RL \geq ARL + 0.5 \times SDRL)$	30.89%	22.62%	22.18%	20.93%	21.34%
1.75	ARL	2.8675	21.6215	41.4495	62.6400	96.9205
	SDRL	1.9585	19.7591	47.2296	81.4611	138.2698
	$p(RL < ARL - 0.5 \times SDRL)$	27.17%	37.74%	41.05%	42.66%	45.16%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	25.44%	25.20%	22.57%	22.45%	20.03%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	17.77%	14.94%	14.29%	13.41%	13.05%
	$p(RL \geq ARL + 0.5 \times SDRL)$	29.62%	22.12%	22.09%	21.48%	21.76%
2	ARL	2.7835	18.9840	37.5205	57.5085	91.9145
	SDRL	1.9871	18.2040	42.5330	75.4716	126.2704
	$p(RL < ARL - 0.5 \times SDRL)$	29.67%	37.20%	41.29%	43.43%	45.37%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	25.98%	25.25%	22.81%	21.28%	20.41%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	17.54%	15.09%	13.36%	13.77%	12.37%
	$p(RL \geq ARL + 0.5 \times SDRL)$	26.81%	22.46%	22.54%	21.52%	21.85%
2.5	ARL	2.4965	16.5355	29.5760	46.0600	78.8660
	SDRL	1.7984	14.4771	35.6754	63.5406	108.0081
	$p(RL < ARL - 0.5 \times SDRL)$	34.73%	36.90%	40.87%	42.48%	44.86%
	$p(ARL - 0.5 \times SDRL \leq RL < ARL)$	26.56%	25.00%	22.92%	22.53%	21.15%
	$p(ARL \leq RL < ARL + 0.5 \times SDRL)$	16.58%	16.61%	13.98%	13.43%	12.31%
	$p(RL \geq ARL + 0.5 \times SDRL)$	22.13%	21.47%	22.23%	21.56%	21.68%

ARL, average run length; SDRL, standard deviation of run length.

control chart in detecting mean shifts in terms of both accuracy and precision criteria. It is also concluded that the ability of MAX-EWMAMS control chart in detecting mean shifts improves as the value of parameter D increases.

Table II shows the performance of the MAX-EWMAMS control chart in detecting mean shifts for different values of parameter C when D is fixed equal to 1. We can see in Table II that increasing the value of parameter C increases undesirable effect of measurement error on both ARL and SDRL values. However, the effect of parameter C on ability of control chart in detecting mean shifts is less than D.

4.2. *Investigating variance shifts*

In Table III, the capability of MAX-EWMAMS control chart in detecting variance shifts in the presence of measurement error with linearly increasing-type variance when $D=0, 1, 2, 3, 4, 5$ and $C=0$ is investigated. It is clear that for all out-of-control scenarios considered in Table III, increasing the parameter D can adversely affect the ability of the control chart in detecting different variance shifts. It is also seen that both ARLs and SDRLs tend to decrease as the magnitude of variance shifts (ψ) increases.

The simulated ARLs, SDRLs, and empirical distribution of run lengths for different values of parameter C when $D=1$ are given in Table IV. Table IV shows that under different variance shifts; increasing the parameter C can affect adversely the detecting capability of MAX-EWMAMS control chart.

Table X. Distribution of run lengths in detecting simultaneous shifts for different values of C when $D=1$ and $\delta=2$

h_m		3.0799	3.1137	3.1509	3.1686	3.1793
ψ		No error	0	1	2	3
1.1	ARL	3.0805	26.0945	28.9770	33.8875	34.7030
	SDRL	1.6964	23.4642	29.0177	32.4754	36.3658
	$p(RL<ARL-0.5\times SDRL)$	43.86%	39.35%	39.07%	38.80%	39.58%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	24.08%	23.68%	23.28%	24.55%	23.31%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	0%	14.37%	15.02%	14.34%	14.35%
	$p(RL\geq ARL+0.5\times SDRL)$	32.06%	22.60%	22.63%	22.31%	22.76%
1.25	ARL	3.0660	25.0530	27.2285	31.7090	32.3590
	SDRL	1.7913	23.1117	28.9717	32.7581	36.1652
	$p(RL<ARL-0.5\times SDRL)$	47.74%	38.27%	37.53%	39.59%	38.82%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	0%	25.23%	25.14%	23.39%	25.03%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	20.61%	13.77%	14.62%	14.21%	14.03%
	$p(RL\geq ARL+0.5\times SDRL)$	31.65%	22.73%	22.71%	22.81%	22.12%
1.5	ARL	2.9855	22.6280	26.2580	27.8820	30.3835
	SDRL	1.9268	20.9672	25.5934	30.0245	34.4406
	$p(RL<ARL-0.5\times SDRL)$	49.47%	37.92%	37.76%	39.58%	38.84%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	0%	25.37%	24.88%	23.69%	24.98%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	19.64%	14.09%	14.37%	13.55%	14.20%
	$p(RL\geq ARL+0.5\times SDRL)$	30.89%	22.62%	22.99%	22.91%	21.98%
1.75	ARL	2.8675	21.6215	22.9465	25.2325	28.5005
	SDRL	1.9585	19.7591	23.4923	26.9386	30.3158
	$p(RL<ARL-0.5\times SDRL)$	27.17%	37.74%	37.34%	38.32%	39.45%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	25.44%	25.20%	25.59%	24.75%	23.71%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	17.77%	14.94%	14.80%	14.62%	15.11%
	$p(RL\geq ARL+0.5\times SDRL)$	29.62%	22.12%	22.27%	22.31%	21.73%
2	ARL	2.7835	18.9840	21.3470	22.7105	24.9060
	SDRL	1.9871	18.2040	20.9696	25.8067	29.3176
	$p(RL<ARL-0.5\times SDRL)$	29.67%	37.20%	36.96%	39.87%	41.07%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	25.98%	25.25%	25.50%	23.45%	21.40%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	17.54%	15.09%	15.24%	14.10%	14.90%
	$p(RL\geq ARL+0.5\times SDRL)$	26.81%	22.46%	22.30%	22.58%	22.63%
2.5	ARL	2.4965	16.5355	17.8860	18.7690	21.2555
	SDRL	1.7984	14.4771	16.7226	19.0667	21.7477
	$p(RL<ARL-0.5\times SDRL)$	34.73%	36.90%	39.79%	40.08%	38.25%
	$p(ARL-0.5\times SDRL\leq RL<ARL)$	26.56%	25.00%	22.40%	21.85%	25.66%
	$p(ARL\leq RL<ARL+0.5\times SDRL)$	16.58%	16.61%	14.33%	15.06%	13.87%
	$p(RL\geq ARL+0.5\times SDRL)$	22.13%	21.47%	23.48%	23.01%	22.22%

ARL, average run length; SDRL, standard deviation of run length.

4.3. *Investigating simultaneous shifts*

In Tables 5–10, the ability of MAX-EWMAMS in the presence of measurement error with linearly increasing-type variance is investigated. Generally, it is observed that in comparison with separate mean and variance shifts, the control chart has better performance in detecting simultaneous shifts in both process mean and variance in terms of both accuracy and precision criteria. Similar to Tables 1–4, the results of Tables 5–10 show that increasing both parameters D and C leads to an undesirable effect on the ability of MAX-EWMAMS control chart in detecting out-of-control shifts. It is also concluded from Tables 5–10 that the effect of parameter C on ability of control chart in detecting simultaneous shifts in both process mean and variance is less than parameter D. In order to investigate the effect of measurement error on the ability of control chart in detecting simultaneous shifts, we fix the magnitude of mean shift (δ) and compute the run length properties under different magnitudes of variance shifts (ψ).

Tables V and VI show the effect of parameters D and C, respectively, on the ability of MAX-EWMAMS control chart in detecting simultaneous mean and variance shifts when $\delta=0.5$. In Table V, the parameter C is fixed to 0, whereas in Table VI, the parameter D is fixed to 1.

The effect of parameters D and C on the ability of MAX-EWMAMS control chart in detecting simultaneous mean and variance shifts when $\delta=1$ is investigated in Tables VII and VIII, respectively. In Table VII, parameter C is fixed to 0, and in Table VIII, parameter D is fixed to 1.

The effect of parameters D and C on the ability of MAX-EWMAMS control chart in detecting simultaneous mean and variance shifts when $\delta=2$ is investigated in Tables IX and X, respectively. In Table IX, parameter C is fixed to 0, and in Table X, parameter D is fixed to 1.

5. Conclusion

In this paper, we incorporated the measurement error in simultaneous monitoring of process mean and variability. We studied the capability of MAX-EWMAMS control chart in detecting mean shifts, variance shifts, and simultaneous shifts in both under linearly increasing error variance. We utilized a numerical example based on simulation and investigated the effect of measurement error with linearly increasing-type variance on MAX-EWMAMS control chart. We found that measurement error with linearly increasing-type variance can seriously decrease the ability of MAX-EWMAMS control chart in detecting different out-of-control scenarios. Investigating the effects of measurement error on artificial neural network-based control schemes can be considered as a future study.

References

1. Bennett CA. Effect of measurement error on chemical process control. *Industrial Quality Control* 1954; **10**(4):17–20.
2. Abraham S. 1977. Control charts and measurement error. Proceedings of Technical Conference of the American Society for Quality Control, vol. 31, 1977; 370–374.
3. Kanazuka T. The effect of measurement error on the power of $\overline{X} - R$ charts. *Journal of Quality Technology* 1986; **18**(2):91–95.
4. Mittag HJ. Measurement error effect on control chart performance. *In Annual Quality Congress Proceedings-American Society for Quality Control* 1995; **66–73**.
5. Mittag HJ, Stemann D. Gauge imprecision effect on the performance of the XS control chart. *Journal of Applied Statistics* 1998; **25**(3):307–317.
6. Linna KW, Woodall WH. Effect of measurement error on Shewhart control charts. *Journal of Quality Technology* 2001; **33**(2):213–222.
7. Linna KW, Woodall WH, Busby KL. The performance of multivariate control charts in the presence of measurement error. *Journal of Quality Technology* 2001; **33**(3):349–355.
8. Stemann D, Weihs C. The EWMA-X-S-control chart and its performance in the case of precise and imprecise data. *Statistical Papers* 2001; **42**(2): 207–223.
9. Maravelakis P, Panaretos J, Psarakis S. EWMA chart and measurement error. *Journal of Applied Statistics* 2004; **31**(4):445–455.
10. Huwang L, Hung Y. Effect of measurement error on monitoring multivariate process variability. *Statistica Sinica* 2007; **17**(2):749.
11. Abbasi SA. On the performance of EWMA chart in the presence of two-component measurement error. *Quality Engineering* 2010; **22**(3):199–213.
12. Costa AF, Castagliola P. Effect of measurement error and autocorrelation on the $\overline{X}$ chart. *Journal of Applied Statistics* 2011; **38**(4):661–673.
13. Scagliarini M. Multivariate process capability using principal component analysis in the presence of measurement errors. *AStA Advances in Statistical Analysis* 2011; **95**(2):113–128.
14. Chakraborty A, Khurshid A. Measurement error effect on the power of control chart for zero-truncated Poisson distribution. *International Journal for Quality Research* 2013; **7**(3):411–419.
15. Haq A, Brown J, Moltchanova E, Al-Omari AI. Effect of measurement error on exponentially weighted moving average control charts under ranked set sampling schemes. *Journal of Statistical Computation and Simulation* 2014; **27**(4):499–514.
16. Ostadsharif Memar A, Niaki STA. The MAX-EWMAMS control chart for joint monitoring of process mean and variance with individual observations. *Quality and Reliability Engineering International* 2011; **27**(4):499–514.
17. MacGregor JF, Harris TJ. The exponentially weighted moving variance. *Journal of Quality Technology* 1993; **25**(2):106–118.
18. Montgomery DC, Keats J, Runger GC, Messina WS. Integrating statistical process control and engineering process control. *Journal of Quality Technology* 1994; **26**(2):79–87.

Authors' biographies

Adel Khati Dizabadi is an MS graduate of Industrial Engineering at the University of Kurdistan. His research interests are statistical quality control and measurement system.

Mahmoud Shahrokhi is an assistant professor at the University of Kurdistan in Iran. His research interests are quality management and risk analysis.

Mohammad Reza Maleki is a PhD student of Industrial Engineering at Shahed University in Iran. He received his BS and MS in Industrial Engineering from Isfahan University of Technology and Shahed University, respectively. His research interests include the following: statistical quality control, change point estimation, measurement system, and artificial neural network.