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Effect of Measurement Error on Joint Monitoring of Process Mean and Variability under Ranked Set Sampling

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In this paper first we apply ranked set sampling (RSS) procedure in joint monitoring of both process mean and variance. Then, we study the effect of measurement error on joint monitoring of process mean and variance when simple random sampling (SRS) as well as RSS procedure is used. The results prove that the measurement error can seriously deteriorate the ability of the control chart in detecting all out-of-control scenarios. The results also represent that using RSS procedure can improve the ability of the monitoring scheme in detecting mean shifts, variance shifts as well as joint shifts either measurement error exists or not. In other words, RSS procedure can reduce the adverse effect of measurement error on detecting ability of joint monitoring scheme. After that, we utilize multiple measurements at each sample point when both SRS and RSS procedures are used. We investigate the effect of parameters in the covariate model thorough a sensitivity analysis. Finally, the applicability of the proposed control charts is illustrated using a real case of the piston rings in an automotive engine manufactured by a forging process. Copyright © 2016 John Wiley & Sons, Ltd.

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1. Introduction

Nowadays quality engineers are interested in developing a single control chart for joint monitoring of both process location and dispersion. The most recent researches in joint monitoring of process mean and variability are addressed as follows: Zhang *et al.*[1] proposed a single control chart based on the combination of exponentially weighted moving average (EWMA) procedure and generalized likelihood ratio (GLR) test statistic for joint monitoring of both the process mean and variance. Zhang *et al.*[2] proposed a new single control chart based on the combination of EWMA control chart and the GLR test for joint monitoring of multivariate process mean and variability. Khoo *et al.*[3] proposed Max-DEWMA control chart for joint monitoring of the process mean and variability based on the extension of single Max-EWMA control chart to a single double EWMA control chart. They showed that the proposed Max-DEWMA control chart outperforms the Max-EWMA control chart in detecting small and moderate shifts in process mean and/or variance. Teh *et al.*[4] introduced a control scheme based on generally weighted moving average control chart, called the Max-GWMA control chart using a single statistic for joint monitoring of the process mean and variance. Ramos *et al.*[5] studied the misleading signals in joint monitoring methods for the mean vector and covariance matrix in the case of multivariate identical independent distributed outputs. Sheu *et al.*[6] proposed maximum chi-square generally weighted moving average (MCSGWMA) control chart based on the combination of two generally weighted moving average (GWMA) control charts into a single one.

Doğu and Kocakoç[7] provided a multivariate joint change point estimation procedure for monitoring both location and dispersion parameters based on the maximum likelihood estimation (MLE) approach. Garthoff *et al.*[8] attempted to monitor jointly the mean vector and the covariance matrix of multivariate nonlinear times series based on several EWMA and CUSUM control charts. Park[9] studied some combined control charts for joint monitoring of the process mean and variance. First, they provided a review study on the existing combined control charts. Then, they proposed control charts based on the union-intersection test for jointly likelihood ratio statistics. Moreover, they adopted the Liptak combining function for another combined control chart. Chowdhury *et al.*[10] studied Shewhart-type control charts based on the Cucconi statistic, called the Shewhart–Cucconi (SC) chart when the normality assumption is violated. They also proposed a diagnostic procedure in order to determine the type of shift in the process when an out-of-control signal is received. Chowdhury *et al.*[11] proposed a single distribution-free cumulative sum (CUSUM) control chart based on the Lepage

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statistic called CUSUM-Lepage control chart. Moreover, McCracken and Chakraborti[12] provided a review paper on the control schemes for joint monitoring of the process mean and variability. They also discussed some of control schemes proposed by different authors for joint monitoring of multivariate processes, autocorrelated data and individual observations. Maleki and Amiri[13] proposed a novel monitoring scheme based on the combination of two multi-layer perceptron (MLP) neural networks for joint monitoring of the process mean and variability with correlated variable and attribute quality characteristics.

In real production systems, process practitioners are faced with the error which is because of the measuring equipment called measurement error. For example, in national cancer institute's OPEN study, one may be interested in measuring the logarithm of dietary protein intake. In this case, the actual value of long-term log-intake is denoted by X. However, the actual value cannot be observed in practice. Instead, the biomarker of log-protein intake, namely urinary nitrogen denoted by Y can be measured (Subar *et al.*[14]).

In recent researches, undesirable effect of error caused by the measuring system is investigated by several researchers. It is shown that the variability of measurement error can lead to deviation of quality characteristics under investigation from their actual values. We document the recent works which have investigated the undesirable effects of such source of uncertainty on different control charts as follows:

Linna and Woodall[15] extended the measurement error model, i.e. $Y = A + BX + \varepsilon$ in order to investigate the performance of $\overline{X}$ and S^2 control charts. Linna *et al.*[16] explored the effect of measurement error on multivariate Shewhart-type control charts. Maravelakis *et al.*[17] considered the measurement error model suggested by Linna and Woodall[15] and investigated the effect of error term on the ability of EWMA control chart for process mean. They showed that in the presence of measurement error, the EWMA control chart loses its efficiency in detecting random shifts in the process mean.

Chakraborty and Khurshid[18] studied the effect of measurement error on the ability of zero truncated Poisson control chart. Haq *et al.*[19] investigated the effect of measurement error on the ability of EWMA control charts in detecting mean shifts based on ranked set sampling (RSS), median RSS (MRSS), imperfect RSS (IRSS) and imperfect MRSS (IMRSS) schemes. They also studied the effect of multiple measurement and non-constant error variance on the performance of EWMA control charts. Hu *et al.*[20] studied the effect of measurement error on the ability of the variable sample size $(VSS) - \overline{X}$ control chart using a linearly covariate error model. They also proposed a methodology in order to select the optimal parameters by considering the measurement error. Noorossana and Zerehsaz[21] studied the effect of classical additive measurement error model on the monitoring of simple linear profiles with random explanatory variable. They showed that the detecting ability of investigated control charts is significantly affected when measurement error is present in the explanatory variable. Khati Dizabadi *et al.*[22] explored the effect of measurement error with linearly increasing-type variance on MAX-EWMAMS control chart.

In joint monitoring of the process mean and variability, process practitioners aim to detect the occurrence of fault in the process as soon as possible. Hence, it would be helpful to enhance the performance of the joint monitoring schemes in detecting various out-of-control situations. For this purpose, in the first contribution we attempt to incorporate RSS scheme into ELR control chart proposed by Zhang *et al.*[1] for joint monitoring of the process mean and variability. In addition, considering the literature, we found that joint monitoring of the process mean and variability in the presence of measurement error is neglected by researchers. Because of the crucial effect of measurement error on the performance of control charts as well as to fill this research gap, in the second contribution we investigate the effect of measurement error on joint monitoring of both process mean and process variance. In the third contribution, we utilize RSS procedure in order to decrease undesirable effect of measurement error on detecting performance of ELR control chart proposed by Zhang *et al.*[1] for joint monitoring of process mean and variance. In fourth contribution of our work, we also use multiple measurements at each sample point as the second remedial approach for covering the measurement error effect. Finally, we investigate the effect of covariate model parameters on detecting ability of ELR control chart.

The rest of this paper is as organized as follows: In Section 2, we express RSS procedure. In Section 3, we introduce the ELR-RSS control chart. In Section 4, we incorporate the measurement error on ELR-SRS and ELR-RSS control charts and proposed two control charts called MELR-SRS and MELR-RSS control charts, respectively. In Section 5, we utilize multiple measurements at each sample point in order to decrease the effect measurement error on MELR-SRS and MELR-RSS control charts. In Section 6, we give a numerical example based on simulation study in order to evaluate the ability of the proposed methods. In Section 7, the application of the proposed control charts is illustrated by a real dataset. Finally, our concluding remarks and recommendations for the future research are provided in Section 8.

2. Mathematical setup and RSS method

Let X is the quality characteristic under study with the probability density function (pdf) of $f(x)$ and the cumulative distribution function (cdf) of $F(x)$. Let also μ_x and σ_X^2 are the mean and variance of X, respectively. Assume $X_1, X_2, \ldots, X_m$ denotes a simple random sample of size m drawn from $f(x)$ and let $X_{(1:m)}, X_{(2:m)}, \ldots, X_{(m:m)}$ be the ordered statistics of the corresponding sample. For ith ordered statistic, $X_{(i:m)} (i = 1, 2, \ldots, m)$ we have:

$$\mu_{X(i:m)} = \int x f_{(i:m)}(x) dx, \tag{1}$$

$$\sigma^2_{x(i:m)} = \int \left(x - \mu_{X(i:m)}\right)^2 f_{(i:m)}(x) dx, \tag{2}$$

where $f_{(i:m)}(x)$ is the pdf of $X_{(i:m)}$ which is given by:

$$f_{(i:m)}(x) = \frac{m!}{(i-1)!(m-i)!}\{F(x)\}^{i-1}\{1-F(x)\}^{m-i}f(x), -\infty < x < \infty. \tag{3}$$

For detailed information see David and Nagaraja.[23]

2.1. Ranked set sampling

In this section the RSS procedure is summarized as follows:

I Select m random samples, each of size m units, from the population.
II Rank the units within each sample with respect to the variable of interest.
III The smallest ranked unit is selected from the first set. Similarly, the second smallest ranked unit is selected from the second set. The procedure continues and the largest ranked unit is selected from the last set.
IV This completes one cycle of a ranked set sample of size m.

Let $\mathbf{X_1}, \mathbf{X_2}, \ldots, \mathbf{X_m}$ be m independent vectors of observations with probability density function $f(x)$ with finite mean μ_X and variance σ_X^2.

Let $X_{11}, X_{12}, \ldots, X_{1m}; X_{21}, X_{22}, \ldots, X_{2m}; \ldots; X_{m1}, X_{m2}, \ldots, X_{mm}$ be m independent simple random samples each of size m.

Let $X_{i(i:m)}$ is the ith order statistic from the ith sample of size m. Hence, the ranked set sample of size m can be denoted by $X_{1(1:m)}$, $X_{2(2:m)}, \ldots, X_{m(m:m)}$. The unbiased estimator of the population mean (see Takahasi and Wakimoto[24]) is defined according to Equation (1):

$$\overline{X}_{RSS} = \frac{1}{m}\sum_{i=1}^{m} X_{i(i:m)}. \tag{4}$$

It can be statistically checked that:

$$E(\overline{X}_{RSS}) = \mu_X. \tag{5}$$

The variance of $\overline{X}_{RSS}$ is given by (see Haq et al.[19]):

$$Var(\overline{X}_{RSS}) = Var(\overline{X}_{SRS}) - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{X(i:m)} - \mu_X\right)^2. \tag{6}$$

Note that, the variance of simple random sampling (SRS) is equivalent to $\frac{\sigma_X^2}{m}$.

3. Proposed ELR-RSS control chart

We assume that the process observations follow normal distribution with mean μ_X and variance σ_X^2. Then, in order to check the stability of the process over time, the following hypothesis tests are considered after standardizing the process observations:

$$H_o : \begin{cases} \mu = 0 \\ \sigma^2 = 1 \end{cases} \text{ and } H_1 : \begin{cases} \mu \neq 0 \\ or \\ \sigma^2 \neq 1 \end{cases}. \tag{7}$$

In this section, we aim to enhance the detecting ability of ELR control chart (introduced by Zhang *et.al.*[1]) by utilizing RSS approach and propose ELR-RSS control chart for joint monitoring of the process mean and variability.

For this purpose, the GLR statistic (Zhang *et al.*[1]) can be computed based on the RSS procedure as follows:

$$LR_{t,RSS} = m\left(\overline{X}_{t,RSS}^2 + S_{t,RSS}^2 - \ln S_{t,RSS}^2 - 1\right), \tag{8}$$

where m is the sample size, $\overline{X}_{t,RSS}^2$ and $S_{t,RSS}^2$ are the sample mean and sample variance, respectively at subgroup $t; t = 1, 2, \ldots$. When $m \to \infty$ then $LR_t \to \chi_2^2$. The terms $\overline{X}_{t,RSS}^2$ and $S_{t,RSS}^2 - \ln S_{t,RSS}^2$ contribute to the changes of the process mean and variance, respectively. Unlike the other test statistics in the literature, LR_t is a likelihood ratio derived under the setting in which the process mean and variance may change, and thus naturally is sensitive to various shift types. For simplicity, the constant term of (-1) and the sample size m in Equation (8) can be ignored. Hence, we will have:

$$LR_{t,RSS} = \left(\overline{X}_{t,RSS}^2 + S_{t,RSS}^2 - \ln S_{t,RSS}^2\right). \tag{9}$$

Next, we incorporate the EWMA procedure to the construction of $LR_{t,RSS}$ statistic in order to detect small or moderate shifts in the mean and variance of the process, effectively. For tth sample; $t = 1, 2, \ldots$ the EWMA-based statistic for monitoring the process mean and variability based on the sample mean $\overline{X}_{t,RSS}$ and sample variance $S_{t,RSS}^2$, respectively, are given by

$$\begin{cases} U_t^{RSS} = \lambda \overline{X}_{t,RSS} + (1-\lambda) U_{t-1}^{RSS} \\ V_t^{RSS} = \lambda S_{t,RSS}^2 + (1-\lambda) V_{t-1}^{RSS} \end{cases}, \tag{10}$$

where, $U_0^{RSS} = 0$, $V_0^{RSS} = 1$ and λ is the smoothing parameter satisfying $0 < \lambda < 1$. To estimate the process variance, we use the moving average estimation of the process mean U_t^{RSS} instead of $\overline{X}_{t,RSS}$. Consequently:

$$S_{t,RSS}^2 = \frac{1}{m} \sum_{i=1}^{m} \left(X_{i(i:m)t} - U_t^{RSS} \right)^2. \tag{11}$$

Finally, we extend the Equation (9) by substituting U_t^{RSS} and V_t^{RSS} instead of statistics for process mean and variance, respectively.

$$ELR_t^{RSS} = \left(U_t^{RSS} \right)^2 + V_t^{RSS} - \ln\left(V_t^{RSS} \right), \quad t = 1, 2, \ldots \tag{12}$$

If $ELR_t^{RSS} > h$, the proposed ELR-RSS control chart triggers an out-of-control alarm, where $h > 0$ is chosen to achieve an specified in-control (IC) average run length (ARL).

4. Effect of measurement error

In this section, we investigate the effect measurement error on the ELR-SRS and ELR-RSS control charts and propose two control charts called MELR-SRS and MELR-RSS control charts, respectively.

4.1. MELR-SRS control chart

Let X be the quality characteristic under investigation which follows a $N\left(\mu_X, \sigma_X^2\right)$ distribution. Because of the uncertainty in measuring system, we are unable to observe X directly. However, it is easy to measure Y that is linearly related to X (see Linna and Woodall[15]) via the following equation:

$$Y = A + BX + \varepsilon, \tag{13}$$

where A and B are known parameters and ε is the random error term which follows a $N\left(0, \sigma_\varepsilon^2\right)$ distribution and is assumed to be independent form X. Therefore, Y follows a normal random variable with mean $A + B\mu_X$ and variance $B^2\sigma_X^2 + \sigma_\varepsilon^2$, i.e. $\tilde{}Y N\left(A + B\mu_X, B^2\sigma_X^2 + \sigma_\varepsilon^2\right)$.

Let $\overline{Y}_t^{SRS} = \frac{1}{m}\sum_{i=1}^{m} Y_{ti}$ be the sample mean at subgroup t; $t = 1, 2, \ldots$, where $Var\left(\overline{Y}_t^{SRS}\right) = \frac{1}{m}\left(B^2\sigma_X^2 + \sigma_\varepsilon^2\right)$. Considering the effect of measurement error on the ELR-SRS control chart, we have:

$$\begin{cases} U_t^{SRS} = \lambda \overline{Y}_{t,SRS} + (1-\lambda) U_{t-1}^{SRS} \\ V_t^{SRS} = \lambda S_{t,SRS}^2 + (1-\lambda) V_{t-1}^{SRS} \end{cases}. \tag{14}$$

Then,

$$MELR_t^{SRS} = \left(U_t^{SRS} \right)^2 + V_t^{SRS} - \ln\left(V_t^{SRS} \right), \qquad t = 1, 2, \ldots \tag{15}$$

where $S_{t,SRS}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(Y_{it} - U_t^{SRS} \right)^2$, $U_0^{SRS} = A + B\mu_X$, $V_0^{SRS} = B^2\sigma_X^2 + \sigma_\varepsilon^2$ and λ is the smoothing parameter satisfying $0 < \lambda < 1$. If $MELR_t^{SRS} > h$, the proposed MELR-SRS control chart triggers an out-of-control alarm, where $h > 0$ is chosen to achieve a specified IC-ARL.

4.2. MELR-RSS control chart

Recall that because of the measurement error, we monitor Y instead of quality characteristic under investigation X. Therefore, we utilize RSS procedure with respect to quality characteristic Y which is related to X according to Equation (13) Here, $\overline{Y}_t^{RSS} = \frac{1}{m}\sum_{i=1}^{m} Y_{i(i:m)t}$, where $Y_{i(i:m)t}$ is the ith ordered statistic from the ith sample at subgroup t; $t = 1, 2, \ldots$. Here, the mean and variance of Y are $A + B\mu_X$ and $B^2\sigma_X^2 + \sigma_\varepsilon^2$, respectively. It can be statistically checked that $\overline{Y}^{RSS}$ follows a normal distribution with the following parameters:

$$\overline{Y}^{RSS} \sim N\left(A + B\mu_X, \left[\frac{1}{m}\left(B^2\sigma_X^2 + \sigma_\varepsilon^2 \right) - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{Y(i:m)} - \mu_Y \right)^2 \right] \right). \tag{16}$$

The MELR-RSS control statistics for monitoring the process mean and variance considering the effect of measurement error will be.

$$\begin{cases} U_t^{RSS} = \lambda \overline{Y}_{t,RSS} + (1-\lambda) U_{t-1}^{RSS} \\ V_t^{RSS} = \lambda S_{t,RSS}^2 + (1-\lambda) V_{t-1}^{RSS} \end{cases}. \tag{17}$$

Then,

$$MELR_t^{RSS} = \left(U_t^{RSS}\right)^2 + V_t^{RSS} - \ln\left(V_t^{RSS}\right), \quad t = 1, 2, \ldots \tag{18}$$

where $S_{t,RSS}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(Y_{i(i:m)t} - U_t^{RSS}\right)^2$, $U_0^{RSS} = A + B\mu_X$, $V_0^{RSS} = B^2\sigma_X^2 + \sigma_\varepsilon^2$ and λ is the smoothing parameter satisfying $0 < \lambda < 1$. If $MELR_t^{RSS} > h$, the proposed MELR-RSS control chart triggers an out-of-control alarm, where $h > 0$ is chosen to achieve a specified IC-ARL.

5. Multiple measurements

Linna and Woodall[15] suggested multiple measurement approach which is also implemented by Maravelakis[25] and Haq *et al.*[19] to reduce the effect of measurement error. Note that, taking several measurements at each possible value of the underlying quality characteristic at subgroup t generally leads to the smaller variance of the error component. In this section, we suggest and utilize multiple measurements on each sample unit for MELR-SRS and MELR-RSS control charts as well. It is worth noting that, despite of reducing the effect of measurement error by using multiple measurements, increasing the number of measurements leads to the additional cost and time.

5.1. MMELR-SRS control chart

In this subsection for each X_i, k measurements are taken, where k is a positive integer. We consider the covariate model $Y_{ij} = A + BX_i + \varepsilon_{ij}$; $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, k$ where Y_{ij} is the ith observation which is measured in jth sequence sampling. For ith observation of tth sample we have:

$$\overline{Y}_{ti} \sim N\left(A + B\mu_X, B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{k}\right), \tag{19}$$

where $\overline{Y}_{ti}$ is the mean value of ith observation which is obtained by k measurements in subgroup t. For sample t; $t = 1, 2, \ldots$ under SRS approach we have:

$$\overline{\overline{Y}}_t^{SRS} \sim N\left(A + B\mu_X, \frac{1}{m}\left(B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{k}\right)\right), \tag{20}$$

where $\overline{\overline{Y}}_t^{SRS} = \frac{1}{m}\sum_{i=1}^{m}\overline{Y}_{ti}$. The plotting statistic of the MELR-SRS control chart based on $\overline{\overline{Y}}_t^{SRS}$ is:

$$\begin{cases} U_t^{SRS} = \lambda \overline{\overline{Y}}_{t,SRS} + (1-\lambda) U_{t-1}^{SRS} \\ V_t^{SRS} = \lambda S_{t,SRS}^2 + (1-\lambda) V_{t-1}^{SRS} \end{cases}. \tag{21}$$

Then:

$$MMELR_t^{SRS} = \left(U_t^{SRS}\right)^2 + V_t^{SRS} - \ln\left(V_t^{SRS}\right), \quad t = 1, 2, \ldots \tag{22}$$

where $S_{t,SRS}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\overline{Y}_{it} - U_t^{SRS}\right)^2$, $U_0^{SRS} = A + B\mu_X$, $V_0^{SRS} = B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{k}$ and λ is the smoothing parameter satisfying $0 < \lambda < 1$. Note that for $k = 1$, the control statistic will be equivalent to that in MELR-SRS control chart given in Section 4-1. Similarly, If $MMELR_t^{SRS} > h$, the proposed MELR-SRS control chart signals an out-of-control alarm, where $h > 0$ is chosen to achieve a desired IC-ARL.

5.2. MMELR-RSS control chart

Under RSS scheme, the overall mean at subgroup t is equal to $\overline{\overline{Y}}_t^{RSS} = \frac{1}{m}\sum_{i=1}^{m}\overline{Y}_{i(i:m)t}$ where:

$$\begin{gathered} \overline{Y}_{i(i:m)tj} = \frac{1}{k}\sum_{j=1}^{k} Y_{ij(i:m)} \\ Var\left(\overline{\overline{Y}}_t^{RSS}\right) = \frac{1}{m}\left(B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{k}\right) - \frac{1}{m^2}\sum_{i=}^{m}\left(\mu_{Y(i:m)t} - \mu_Y\right)^2. \end{gathered} \tag{23}$$

The control statistic of MMELR-RSS control chart based on $\overline{\overline{Y}}_t^{RSS}$ is:

$$\begin{cases} U_t^{RSS} = \lambda \overline{\overline{Y}}_{t,RSS} + (1-\lambda) U_{t-1}^{RSS} \\ V_t^{RSS} = \lambda S_{t,RSS}^2 + (1-\lambda) V_{t-1}^{RSS} \end{cases} \tag{24}$$

Then:

$$MMELR_t^{RSS} = \left(U_t^{RSS}\right)^2 + V_t^{RSS} - \ln\left(V_t^{RSS}\right), \quad t = 1, 2, \ldots \tag{25}$$

where $S_{t,RSS}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\overline{Y}_{i(i:m)t} - U_t^{RSS}\right)^2$, $U_0^{RSS} = A + B\mu_X$, $V_0^{RSS} = B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{k}$ and λ is the smoothing parameter ($0 < \lambda < 1$). If $MMELR_t^{RSS} > h$, the proposed MELR-SRS control chart shows an out-of-control alarm, where $h > 0$ is selected to obtain a specified IC ARL.

Table I. The ARL and SDRL values of the ELR-SRS, ELR-RSS control charts with error and without error for $m = 5$, $\lambda = 0.2$, $B = 1$, $A = 0$ and IC ARL = 370

			No error		$\sigma_\varepsilon = 0.2$		$\sigma_\varepsilon = 0.3$		$\sigma_\varepsilon = 0.4$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.2421	1.1534	1.2471	1.1615	1.2638	1.1857	1.3015	1.2298
0.00	1.00	ARL	370.78	369.67	369.12	371.10	369.52	369.43	371.91	368.86
		SDRL	366.15	363.19	368.10	369.19	368.77	367.87	367.68	365.59
	0.75	ARL	18.00	12.20	24.18	16.61	41.06	30.49	129.01	122.52
		SDRL	10.36	6.13	15.63	9.55	31.63	21.67	115.78	110.08
	0.50	ARL	6.05	4.66	6.97	5.47	8.65	7.07	12.93	10.87
		SDRL	1.00	0.71	1.35	0.96	2.09	1.50	4.46	3.26
	0.25	ARL	4.14	3.37	4.99	4.00	5.70	4.88	7.44	6.48
		SDRL	0.34	0.28	0.20	0.09	0.58	0.42	1.01	0.75
0.25	1.00	ARL	37.36	29.80	38.10	32.35	42.23	44.04	58.13	57.65
		SDRL	32.42	22.63	32.12	25.46	36.93	37.41	52.26	51.72
	0.75	ARL	12.79	8.88	15.23	11.02	20.63	16.43	38.08	36.96
		SDRL	6.12	3.44	8.00	4.73	12.32	8.74	28.28	26.27
	0.50	ARL	5.75	4.44	6.50	5.12	7.87	6.41	10.88	9.28
		SDRL	0.87	0.61	1.15	0.81	1.70	1.18	3.24	2.32
	0.25	ARL	4.06	3.20	4.90	3.98	5.48	4.68	7.04	6.16
		SDRL	0.24	0.40	0.31	0.05	0.54	0.48	0.88	0.68
0.50	1.00	ARL	9.71	6.64	9.81	6.85	10.41	7.79	11.81	9.51
		SDRL	5.44	2.63	5.56	2.81	6.14	3.43	7.57	4.71
	0.75	ARL	7.33	5.39	8.02	5.94	9.15	7.11	11.47	9.54
		SDRL	2.43	1.27	2.83	1.50	3.62	2.00	5.34	3.13
	0.50	ARL	5.00	3.94	5.46	4.31	6.20	5.12	7.79	6.62
		SDRL	0.67	0.43	0.82	0.52	1.11	0.72	1.72	1.13
	0.25	ARL	3.99	3.00	4.35	3.80	5.05	4.10	6.10	5.29
		SDRL	0.03	0.06	0.47	0.39	0.35	0.30	0.68	0.49
0.75	1.00	ARL	5.10	3.69	5.18	3.79	5.47	4.16	5.82	4.59
		SDRL	2.08	1.05	2.25	1.13	2.35	1.28	2.76	1.56
	0.75	ARL	4.87	3.64	5.06	3.83	5.40	4.13	6.16	5.01
		SDRL	1.24	0.67	1.39	0.75	1.63	0.87	2.08	1.14
	0.50	ARL	4.15	3.15	4.37	3.45	4.75	3.93	5.51	4.67
		SDRL	0.51	0.36	0.63	0.50	0.78	0.47	1.07	0.67
	0.25	ARL	3.93	3.00	3.99	3.00	4.26	3.72	4.98	4.24
		SDRL	0.17	0.00	0.13	0.09	0.45	0.44	0.57	0.43
1.00	1.00	ARL	3.50	2.59	3.60	2.62	3.77	2.86	3.76	3.03
		SDRL	1.22	0.66	1.27	0.70	1.34	0.76	1.51	0.88
	0.75	ARL	3.49	2.71	3.57	2.79	3.75	2.92	4.04	3.33
		SDRL	0.85	0.51	0.91	0.53	1.02	0.59	1.21	0.71
	0.50	ARL	3.34	2.78	3.45	2.86	3.66	3.00	4.02	3.40
		SDRL	0.50	0.41	0.56	0.34	0.63	0.32	0.79	0.53
	0.25	ARL	3.03	2.86	3.23	2.95	3.58	2.99	3.94	3.31
		SDRL	0.17	0.34	0.42	0.2	0.49	0.07	0.51	0.46

Table II. The ARL and SDRL values of the MELR-SRS, MELR-RSS control charts for different values λ when $m=5$, $\sigma_\varepsilon=0.20$, $A=0$, $B=1$ and IC $ARL=370$

			$\lambda=0.1$		$\lambda=0.2$		$\lambda=0.25$		$\lambda=0.3$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.1088	1.0706	1.2471	1.1615	1.3235	1.2120	1.4035	1.2645
0.00	0.25	ARL	6.30	5.08	4.99	4.00	4.39	3.74	4.12	3.20
		SDRL	0.46	0.27	0.20	0.09	0.49	0.43	0.32	0.40
	0.50	ARL	8.57	6.92	6.97	5.47	6.68	5.16	6.53	4.96
		SDRL	1.11	0.84	1.35	0.96	1.55	1.03	1.78	1.13
	0.75	ARL	20.87	15.79	24.18	16.61	27.97	18.44	32.55	20.26
		SDRL	8.92	5.97	15.63	9.55	20.16	12.19	25.86	14.63
	1.00	ARL	369.12	369.23	369.12	371.10	370.03	369.60	368.98	370.60
		SDRL	363.25	363.74	367.10	369.19	363.64	369.21	365.34	369.03
	1.25	ARL	15.25	10.85	18.10	11.80	19.71	12.63	21.79	13.60
		SDRL	10.46	6.90	15.18	9.06	17.24	10.21	19.73	11.49
	1.50	ARL	5.88	4.46	5.73	4.19	5.87	4.17	6.02	4.26
		SDRL	3.29	2.34	3.74	2.52	4.14	2.64	4.44	2.88
	1.75	ARL	3.52	2.80	3.32	2.56	3.31	2.52	3.34	2.51
		SDRL	1.84	1.36	1.93	1.38	2.01	1.43	2.26	1.47
0.25	0.25	ARL	6.15	5.02	4.90	3.98	4.24	3.55	4.06	3.10
		SDRL	0.35	0.15	0.31	0.05	0.43	0.49	0.25	0.31
	0.50	ARL	8.05	6.53	6.50	5.12	6.16	4.80	6.00	4.59
		SDRL	0.96	0.70	1.15	0.81	1.30	0.86	1.49	0.94
	0.75	ARL	14.89	11.47	15.23	11.02	16.60	11.62	18.37	12.56
		SDRL	5.05	3.10	8.00	4.73	10.13	6.01	12.51	7.39
	1.00	ARL	29.77	21.49	38.10	32.35	44.29	41.27	51.12	50.68
		SDRL	20.66	11.96	32.12	25.46	39.56	35.13	44.84	47.46
	1.25	ARL	11.19	8.05	12.32	8.38	13.15	8.87	13.92	9.50
		SDRL	6.91	4.41	9,28	5.51	10.58	6.37	11.43	7.24
	1.50	ARL	5.38	4.07	5.25	3.80	5.29	3.81	5.45	3.85
		SDRL	2.90	1.99	3.38	2.16	3.58	2.28	3.81	2.47
	1.75	ARL	3.40	2.68	3.20	2.45	3.19	2.42	3.21	2.37
		SDRL	1.75	1.30	1.82	1.29	1.99	1.33	2.05	1.34
	0.25	ARL	5.97	4.99	4.35	3.80	4.03	3.07	3.94	3.00
		SDRL	0.18	0.10	0.47	0.39	0.17	0.26	0.25	0.08
	0.50	ARL	6.88	5.53	5.46	4.31	5.12	4.03	4.90	3.76
		SDRL	0.77	0.55	0.82	0.52	0.90	0.56	0.97	0.64
	0.75	ARL	8.94	6.93	8.02	5.94	7.95	5.83	8.18	5.81
		SDRL	2.22	1.25	2.83	1.50	3.18	1.70	3.87	2.01
0.50	1.00	ARL	9.89	7.20	9.81	6.85	10.28	7.11	10.92	7.58
		SDRL	4.43	2.23	5.56	2.81	6.50	3.31	7.59	4.07
	1.25	ARL	6.89	5.00	6.75	4.73	6.93	4.83	7.19	4.88
		SDRL	3.58	2.04	4.17	2.30	4.53	2.55	5.04	2.80
	1.50	ARL	4.42	3.34	4.19	3.06	4.19	3.04	4.23	2.99
		SDRL	2.26	1.46	2.48	1.53	2.60	1.58	2.77	1.61
	1.75	ARL	3.15	2.42	2.88	2.21	2.87	2.16	2.86	2.12
		SDRL	1.57	1.08	1.60	1.10	1.67	1.07	1.75	1.12
	0.25	ARL	5.05	4.00	3.99	3.00	3.73	3.00	3.23	2.98
		SDRL	0.25	0.09	0.13	0.09	0.44	0.02	0.42	0.12
	0.50	ARL	5.52	4.39	4.37	3.45	4.09	3.16	3.87	3.05
		SDRL	0.68	0.49	0.63	0.50	0.63	0.38	0.68	0.32
	0.75	ARL	5.92	4.60	5.06	3.83	4.87	3.65	4.74	3.51
		SDRL	1.38	0.76	1.39	0.75	1.46	0.77	1.58	0.81
0.75	1.00	ARL	5.68	4.28	5.18	3.79	5.10	3.67	5.12	3.62
		SDRL	2.07	1.13	2.25	1.13	2.33	1.19	2.55	1.27
	1.25	ARL	4.62	3.47	4.29	3.10	4.27	3.04	4.30	3.02
		SDRL	2.06	1.20	2.20	1.22	2.28	1.22	2.50	1.29
	1.50	ARL	3.55	2.66	3.26	2.41	3.23	2.36	3.20	2.31

(*Continues*)

Table II. *Continued.*

			$\lambda=0.1$		$\lambda=0.2$		$\lambda=0.25$		$\lambda=0.3$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.1088	1.0706	1.2471	1.1615	1.3235	1.2120	1.4035	1.2645
		SDRL	1.68	1.06	1.72	1.04	1.81	1.07	1.88	1.07
	1.75	ARL	2.72	2.11	2.51	1.92	2.48	1.87	2.45	1.83
		SDRL	1.31	0.88	1.34	0.86	1.36	0.86	1.38	0.87
	0.25	ARL	4.16	3.27	3.23	2.95	3.01	2.49	2.97	2.05
		SDRL	0.38	0.44	0.42	0.2	0.14	0.50	0.15	0.22
	0.50	ARL	4.28	3.39	3.45	2.86	3.24	2.61	3.08	2.36
		SDRL	0.63	0.49	0.56	0.34	0.52	0.48	0.48	0.48
	0.75	ARL	4.23	3.30	3.57	2.79	3.42	2.63	3.28	2.50
		SDRL	0.98	0.58	0.91	0.53	0.91	0.55	0.92	0.55
1.00	1.00	ARL	3.92	3.03	3.60	2.62	3.38	2.52	3.31	2.43
		SDRL	1.28	0.75	1.27	0.70	1.31	0.69	1.35	0.70
	1.25	ARL	3.42	2.61	3.11	2.31	3.03	2.23	2.98	2.17
		SDRL	1.41	0.81	1.43	0.77	1.43	0.79	1.45	0.78
	1.50	ARL	2.83	2.17	2.60	1.96	2.56	1.90	2.52	1.84
		SDRL	1.27	0.78	1.28	0.76	1.31	0.75	1.35	0.76
	1.75	ARL	2.37	1.86	2.16	1.65	2.12	1.61	2.10	1.58
		SDRL	1.09	0.73	1.10	0.70	1.22	0.68	1.12	0.68

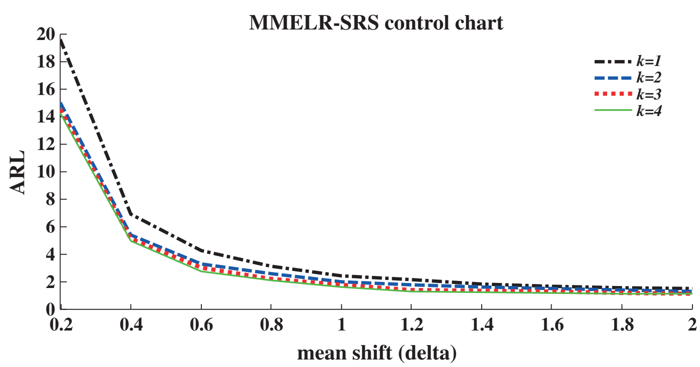

Figure 1. The ARLs of MMELR-SRS chart under mean shifts for multiple measurements when $k=1$, $k=2$, $k=3$, $k=4$, $m=5$, $A=0$, $B=1$, $\sigma_\varepsilon=0.30$ and IC ARL = 200

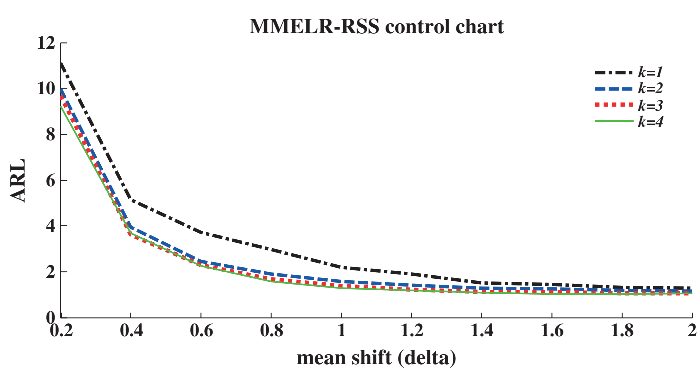

Figure 2. The ARLs of MMELR-RSS chart under mean shifts for multiple measurements when $k=1$, $k=2$, $k=3$, $k=4$, $m=5$, $A=0$, $B=1$, $\sigma_\varepsilon=0.30$ and IC ARL = 200

6. Performance evaluation

In this section we present a numerical example based on simulation study in order to investigate the ability of the proposed control charts in detecting different out-of-control scenarios. Note that, there are different approaches for calculating the ARLs in the

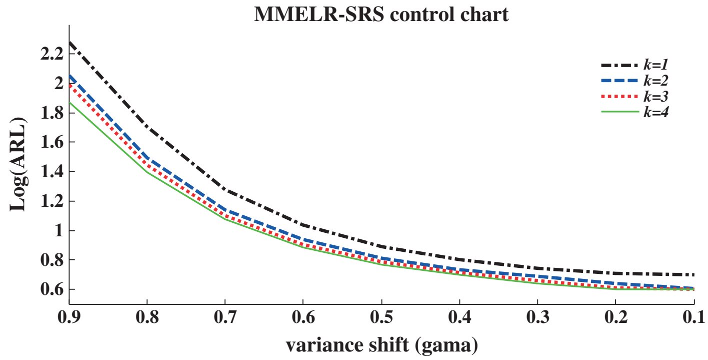

Figure 3. The Log (ARLs) of MMELR-SRS chart under variance shifts for multiple measurements when $k = 1$, $k = 2$, $k = 3$, $k = 4$, $m = 5$, $A = 0$, $B = 1$, $\sigma_\varepsilon = 0.30$ and IC ARL = 200

literature such as Markov chains, integral equations and Monte Carlo simulations. In this paper, Monte Carlo simulation is used in order to calculate the ARLs in all proposed control charts. In the presented numerical example, we suppose that X follows standard normal distribution. The results of simulation study in terms of two criteria including the ARL and standard deviation of run lengths (SDRL) which is obtained by 10 000 replicates are summarized in Table I. Note that, the out-of-control conditions for the process mean and variance are denoted by $\mu_1 = \mu_0 + \delta\sigma_0$ and $\sigma_1 = \gamma\sigma_0$, respectively. It is worth to mention that μ_0 and σ_0 are the mean and standard deviation of X, respectively. Table I shows the ARLs and SDRLs obtained by proposed ELR-SRS, ELR-RSS, MELR-SRS and MELR-RSS control charts under different mean shifts, variance shifts as well as joint shifts in both mean and variance. By comparing SRS and RSS schemes without measurement error, it is concluded that utilizing RSS scheme can improve the detecting ability of the ELR control chart rather than SRS scheme. It is also seen that measurement error can deteriorate the detecting ability of ELR-SRS control chart in detecting different out-of-control scenarios. As the variance of measurement error increases, the ARLs and SDRLs increase. We can also see the effect of measurement error on ELR-RSS control chart under different values of σ_ε. We can observe that measurement error affects adversely the detecting ability of ELR-RSS control chart. Table I proves that in the presence of measurement error, using RSS approach leads to smaller ARLs and SDRLs in comparison with SRS approach. This fact means that RSS can be applied as an effective procedure for reducing the undesirable effect of the measurement error. Note that Table I contains decreasing shifts in the variability; similar results (not reported here) are obtained for increasing shifts in the variability.

Here, a sensitivity analysis with respect to parameter λ is provided, and the results are summarized in Table II. It is observed in Table II that as parameter λ increases, the ability of control chart under both sampling strategies in detecting large shifts improves. In contrast, the ability of the control charts in detecting small shifts under both sampling strategies decreases as the parameter λ increases.

The ARLs obtained by using multiple measurements at each sample point are summarized in Figures 1 and 2. The results show that utilizing multiple measurement approach can improve the ability of both MELR-SRS and MELR-RSS control charts in detecting different mean shifts. The ARLs obtained by using multiple measurements approach in detecting different variance shifts are depicted in Figures 3 and 4. The results of Figures 3 and 4 show that utilizing multiple measurement approach can improve the ability of both control charts under different variance shifts. Considering Tables I and II as well as Figures 1, 2, 3 and 4 we can conclude that joint utilizing of both proposed remedial approaches including RSS and multiple measurements leads to the best performance of control chart in detecting separate mean and variance shifts as well as joint shifts in both process mean and variance.

Here, we provide a sensitivity analysis on the value of parameter B and summarize the results in Table III. The results show that as the parameter B increases, the effect of measurement error on detecting ability of ELR-SRS and ELR-RSS control charts in terms of

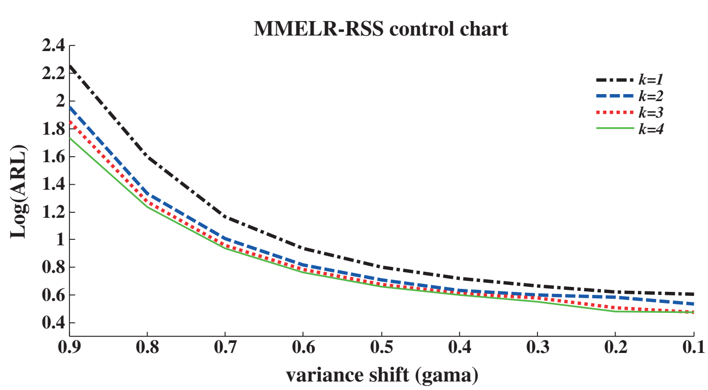

Figure 4. The Log (ARLs) of MMELR-RSS chart under variance shifts for multiple measurements when $k = 1$, $k = 2$, $k = 3$, $k = 4$, $m = 5$, $A = 0$, $B = 1$, $\sigma_\varepsilon = 0.30$ and IC ARL = 200

Table III. The ARL and SDRL values of the MELR-SRS and MELR-RSS charts for different values B when $m=5$, $\lambda=0.2$, $\sigma_\varepsilon=0.2$, $A=0$ and IC ARL = 370

			$B=1$		$B=2$		$B=3$		$B=4$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.2471	1.1615	4.6673	4.4895	11.9235	11.5210	22.5824	21.8896
0.00	1.00	ARL	370.12	369.33	369.29	368.97	369.27	371.19	368.95	368.77
		SDRL	367.10	368.19	360.13	361.37	365.16	368.51	370.20	372.31
	1.25	ARL	17.75	12.05	12.42	10.59	12.32	10.41	12.08	10.39
		SDRL	14.95	9.08	9.92	8.01	9.81	7.88	9.78	7.77
	1.50	ARL	5.72	4.22	4.48	3.89	4.41	3.80	4.43	3.86
		SDRL	3.76	2.54	2.91	2.35	2.88	2.35	2.94	2.34
	1.75	ARL	3.32	2.57	2.74	2.42	2.69	2.40	2.71	2.39
		SDRL	1.94	1.38	1.59	1.33	1.62	1.32	1.60	1.30
0.25	1.00	ARL	37.74	32.35	144.12	166.56	159.85	183.29	162.10	185.89
		SDRL	32.00	25.46	143.18	163.91	157.92	177.92	159.61	184.52
	1.25	ARL	12.22	7.56	10.44	9.06	10.51	8.93	10.40	8.96
		SDRL	9.14	5.57	7.98	6.53	8.20	6.45	8.18	6.51
	1.50	ARL	5.19	3.83	4.28	3.68	4.20	3.59	4.19	3.66
		SDRL	3.32	2.18	2.70	2.14	2.72	2.21	2.69	2.15
	1.75	ARL	3.20	2.46	2.77	2.35	2.68	2.31	2.61	2.33
		SDRL	1.84	1.30	1.54	1.26	1.58	1.22	1.50	1.24
0.50	1.00	ARL	9.87	6.90	30.02	31.20	35.58	38.44	37.07	40.78
		SDRL	5.63	2.81	25.88	25.73	31.99	34.19	33.51	36.05
	1.25	ARL	6.82	4.87	7.09	6.14	7.30	6.32	7.46	6.44
		SDRL	4.17	2.35	4.90	3.73	5.16	3.96	5.29	3.98
	1.50	ARL	4.19	3.20	3.69	3.16	3.66	3.14	3.65	3.22
		SDRL	2.48	1.53	2.28	1.70	2.25	1.78	2.26	1.76
	1.75	ARL	2.88	2.25	2.51	2.19	2.49	2.17	2.45	2.18
		SDRL	1.59	1.19	1.42	1.10	1.43	1.12	1.41	1.10
0.75	1.00	ARL	5.17	3.79	9.75	8.97	11.25	10.36	11.61	10.75
		SDRL	2.24	1.13	6.43	5.05	8.05	4.47	8.36	6.84
	1.25	ARL	4.29	3.15	4.75	4.14	4.98	4.27	4.97	4.33
		SDRL	2,23	1.21	2.86	2.11	3.05	2.25	3.19	2.31
	1.50	ARL	3.27	2.41	3.04	2.64	3.09	2.66	3.05	2.67
		SDRL	1.75	1.02	1.78	1.33	1.82	1.34	1.80	1.34
	1.75	ARL	2.50	1.92	2.25	1.95	2.22	1.98	2.22	1.94
		SDRL	1.32	0.87	1.24	0.94	1.23	0.97	1.22	0.94
1.00	1.00	ARL	3.47	2.62	5.04	4.44	5.52	4.88	5.57	4.99
		SDRL	1.27	0.70	2.69	1.82	3.10	2.16	3.20	2.24
	1.25	ARL	3.11	2.30	3.40	2.94	3.48	3.01	3.49	3.04
		SDRL	1.39	0.77	1.85	1.29	1.94	1.35	1.96	1.39
	1.50	ARL	2.50	1.96	2.53	2.16	2.58	2.21	2.54	2.20
		SDRL	1.26	0.76	1.35	0.97	1.39	1.01	1.42	1.02
	1.75	ARL	2.16	1.66	1.98	1.74	2.00	1.79	2.01	1.73
		SDRL	1.09	0.68	1.04	0.78	1.06	0.79	1.07	0.78

both ARL and SDRL criteria under mean shifts increases. However, we cannot determine the optimal value of parameter B for ELR-SRS and ELR-RSS control charts under variance shifts and joint shifts in the process mean and variability. It is worth to mention that Table III contains increasing shifts in the variability; similar results (not reported here) are obtained for decreasing shifts in the variability. The effect of sample size on MELR-SRS and MELR-RSS control charts when $\lambda=0.2, \sigma_\varepsilon=0.2, A=0, B=1$ and $ARL_0=370$ is evaluated in Table IV. Table IV represents that as the sample size increases, the effect of measurement error on ARLs and SDRLs decreases.

The effect of parameter A on MELR-SRS and MELR-RSS control charts under various mean shifts when $B=1$ are shown in Figure 5. The results show that in both MELR-SRS and MELR-RSS control charts, selecting $A\neq 0$ leads to the smaller ARLs in comparison with $A=0$. However, the ability of MELR-SRS and MELR-RSS control charts are almost the same when $A=1, A=2$ and $A=3$. The effect of parameter A on MELR-SRS and MELR-RSS control charts under various mean shifts when $B=2$ is also investigated in Figure 6. The

Table IV. The ARL and SDRL values of the MELR-SRS, MELR-RSS control charts for different sample sizes m when $\lambda = 0.2$, $\sigma_\varepsilon = 0.20$, $A = 0$, $B = 1$ and IC ARL = 370

			$m = 5$		$m = 10$		$m = 15$		$m = 20$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.2471	1.1615	1.1263	1.0626	1.0858	1.0360	1.0652	1.0244
0.00	0.25	ARL	4.98	4.00	3.51	2.97	3.00	2.00	2.99	2.00
		SDRL	0.20	0.08	0.49	0.14	0.00	0.00	0.03	0.00
	0.5	ARL	6.97	5.48	4.68	3.28	3.88	2.78	3.31	2.05
		SDRL	1.32	0.95	0.67	0.45	0.49	0.41	0.46	0.21
	0.75	ARL	24.43	16.55	12.63	7.54	9.13	5.25	7.46	4.21
		SDRL	15.97	9.51	6.03	2.63	3.51	1.33	2.49	0.89
	1.00	ARL	369.12	371.10	371.43	368.97	371.86	369.15	370.82	372.11
		SDRL	367.10	369.19	368.82	369.76	369.67	371.22	372.65	371.19
	1.25	ARL	18.24	11.96	9.32	5.28	6.61	3.46	5.26	2.65
		SDRL	15.09	9.22	6.45	3.02	3.99	1.61	2.93	1.06
	1.50	ARL	5.80	4.20	3.42	2.23	2.63	1.59	2.22	1.29
		SDRL	3.79	2.55	1.78	0.98	1.23	0.62	0.95	0.46
	1.75	ARL	3.32	2.59	2.11	1.47	1.69	1.13	1.46	1.01
		SDRL	1.90	1.39	1.00	0.59	0.72	0.34	0.59	0.13
0.25	0.25	ARL	4.90	4.00	3.27	2.81	3.00	2.00	2.95	2.00
		SDRL	0.31	4.72	0.44	0.39	0.00	0.00	0.20	0.00
	0.5	ARL	6.48	5.12	4.40	3.10	3.62	2.45	3.12	2.05
		SDRL	1.16	0.81	0.57	0.3	0.51	0.49	0.32	0.05
	0.75	ARL	15.11	10.98	8.59	5.36	4.66	3.88	5.41	3.18
		SDRL	7.69	4.72	3.15	1.28	1.90	0.71	1.42	0.44
	1.00	ARL	38.75	32.84	19.52	10.56	13.27	6.31	10.30	4.57
		SDRL	33.12	25.85	14.18	5.17	8.55	2.15	6.07	1.20
	1.25	ARL	12.19	8.32	6.61	3.93	4.88	1.11	3.97	2.08
		SDRL	9.15	5.51	4.00	1.84	2.64	0.31	1.93	0.67
	1.50	ARL	5.23	3.81	3.15	2.05	2.44	1.51	2.05	1.20
		SDRL	3.33	2.16	1.61	0.85	1.11	0.57	0.87	0.40
	1.75	ARL	3.19	2.45	2.06	1.42	1.65	2.66	1.42	1.01
		SDRL	1.83	1.30	0.97	0.57	0.70	0.98	0.56	0.10
0.50	0.25	ARL	4.35	3.80	3.00	2.01	2.99	2.00	2.09	2.00
		SDRL	0.47	0.39	0.08	0.13	0.05	0.00	0.28	0.00
	0.5	ARL	5.45	4.31	3.78	2.91	3.07	2.18	2.89	2.00
		SDRL	0.82	0.52	0.50	0.28	0.27	0.04	0.30	0.00
	0.75	ARL	7.98	5.97	4.99	3.30	3.93	2.50	3.39	2.02
		SDRL	2.82	1.52	1.28	0.51	0.87	0.50	0.69	0.12
	1.00	ARL	9.72	6.91	5.73	3.42	4.33	2.50	3.63	2.01
		SDRL	5.57	2.86	2.55	0.88	1.67	0.55	1.27	0.29
	1.25	ARL	6.78	4.76	4.04	2.48	3.11	1.84	2.60	1.49
		SDRL	4.25	2.37	1.99	0.82	1.35	0.53	1.02	0.50
	1.50	ARL	4.18	3.06	2.62	1.73	2.06	1.27	1.77	1.05
		SDRL	2.46	1.54	1.24	0.64	0.87	0.44	0.70	0.23
	1.75	ARL	2.89	2.20	1.89	1.31	1.53	1.04	1.33	1.00
		SDRL	1.62	1.08	0.86	0.49	0.63	0.20	0.51	0.04
0.75	0.25	ARL	3.99	3.00	2.99	2.00	2.10	2.00	2.00	1.15
		SDRL	0.12	0.08	0.05	0.00	0.30	0.00	0.00	0.35
	0.5	ARL	4.37	3.45	3.04	2.03	2.56	2.00	2.07	1.64
		SDRL	0.63	0.50	0.32	0.11	0.49	0.00	0.27	0.45
	0.75	ARL	5.09	3.83	3.32	2.20	2.70	2.00	2.34	1.68
		SDRL	1.39	0.74	0.75	0.40	0.57	0.13	0.49	0.47
	1.00	ARL	5.13	3.76	3.29	2.14	2.63	1.76	2.25	1.27
		SDRL	2.18	1.14	1.13	0.45	0.80	0.42	0.64	0.44
	1.25	ARL	4.31	3.12	2.76	1.81	2.18	1.32	1.86	1.03
		SDRL	2.21	1.21	1.16	0.52	0.80	0.46	0.66	0.18
	1.50	ARL	3.27	2.40	2.13	1.41	1.70	1.06	1.48	1.00

(*Continues*)

Table IV. *Continued.*

			$m=5$		$m=10$		$m=15$		$m=20$	
			SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
δ	γ	h	1.2471	1.1615	1.1263	1.0626	1.0858	1.0360	1.0652	1.0244
		SDRL	1.76	1.04	0.94	0.51	0.68	0.25	0.57	0.03
	1.75	ARL	2.50	1.90	1.69	1.17	1.37	1.00	1.20	1.00
		SDRL	1.33	0.86	0.74	0.38	0.54	0.09	0.41	0.00
1.00	0.25	ARL	3.24	2.86	2.23	2.00	2.00	1.03	1.87	1.00
		SDRL	0.42	0.19	0.42	0.00	0	0.17	0.35	0.00
	0.5	ARL	3.48	2.96	2.41	1.99	2.00	1.15	1.97	1.00
		SDRL	0.56	0.34	0.49	0.06	0.11	0.37	0.15	0.00
	0.75	ARL	3.60	3.19	2.45	1.91	2.03	1.17	1.99	1.00
		SDRL	0.92	0.53	0.56	0.27	0.37	0.38	0.01	0.00
	1.00	ARL	3.47	2.61	2.35	1.67	1.93	1.07	1.68	1.00
		SDRL	1.28	0.70	0.71	0.46	0.53	0.26	0.50	0.00
	1.25	ARL	3.08	2.30	2.05	1.39	1.68	1.02	1.45	1.00
		SDRL	1.39	0.78	0.77	0.48	0.60	0.13	0.52	0.00
	1.50	ARL	2.59	1.96	1.75	1.16	1.43	1.00	1.24	1.00
		SDRL	1.28	0.76	0.72	0.37	0.55	0.05	0.43	0.00
	1.75	ARL	2.16	1.65	1.49	1.06	1.24	1.00	1.11	1.00
		SDRL	1.10	0.76	0.62	0.24	0.44	0.02	0.31	0.00
1.5	0.25	ARL	2.11	1.89	1.93	1.00	1.16	1.00	1.00	1.00
		SDRL	0.31	0.01	0.25	0.00	0.37	0.00	0.02	0.00
	0.5	ARL	2.20	1.92	1.75	1.00	1.21	1.00	1.01	1.00
		SDRL	0.42	0.17	0.42	0.00	0.41	0.00	0.12	0.00
	0.75	ARL	2.18	1.99	1.61	1.00	1.22	1.00	1.04	1.00
		SDRL	0.54	0.37	0.48	0.04	1.41	0.00	0.20	0.00
	1.00	ARL	2.09	1.69	1.48	1.00	1.18	1.00	1.05	1.00
		SDRL	0.68	0.48	0.51	0.06	0.38	0.00	0.22	0.00
	1.25	ARL	1.95	1.53	1.37	1.00	1.13	1.00	1.04	1.00
		SDRL	0.76	0.52	0.50	0.05	0.34	0.00	0.19	0.00
	1.50	ARL	1.79	1.38	1.28	1.00	1.09	1.00	1.02	1.00
		SDRL	0.78	0.50	0.47	0.05	0.29	0.00	0.15	0.00
	1.75	ARL	1.66	1.27	1.20	1.00	1.06	1.00	1.01	1.00
		SDRL	0.74	0.46	0.42	0.03	0.24	0.00	0.11	0.00

results show that the ability of MELR-SRS and MELR-RSS control charts in detecting separate mean shifts improves as the value of A increases. Figures 7 and 8 show the effect of parameter A on MELR-SRS and MELR-RSS control charts under different variance shifts when $B=1$ and $B=2$, respectively. Figures 7 and 8 show that despite of mean shifts, selecting $A=0$ when $B=1$ leads to the best ability of both control charts under shifts in variance. In other words, the ability of both control charts decreases as the value of parameter A increases. Note that, we cannot determine the best value of parameter A in both control charts when simultaneous shifts are occurred.

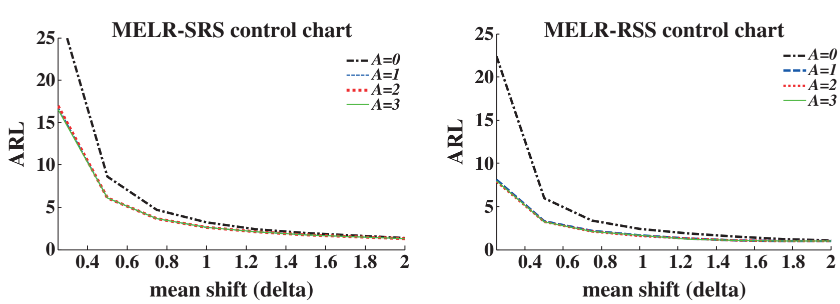

Figure 5. Values of ARL obtained by MELR-SRS and MELR-RSS control charts under mean shift for different values of A when $m=5$, $\lambda=0.20$, $\sigma_\varepsilon=0.2$, $B=1$ and IC ARL = 200

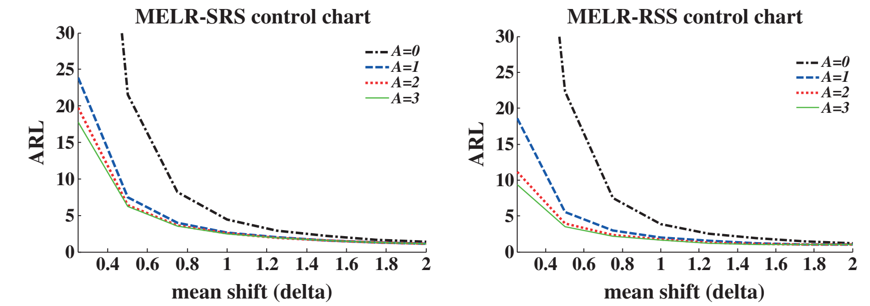

Figure 6. Values of ARL obtained by MELR-SRS and MELR-RSS control charts under mean shift for different values of A when $m = 5$, $\lambda = 0.2$, $\sigma_{\varepsilon} = 0.2$, $B = 2$ and IC ARL = 200

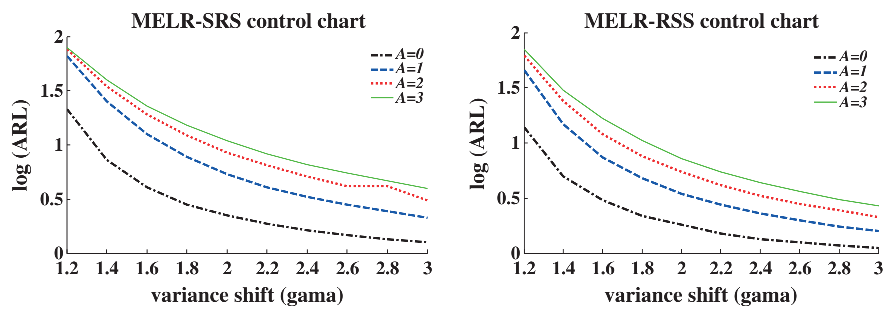

Figure 7. Values of ARL obtained by MELR-SRS and MELR-RSS control charts under variance shift for different values of A when $m = 5$, $\lambda = 0.20$, $\sigma_{\varepsilon} = 0.2$, $B = 1$ and IC ARL = 200

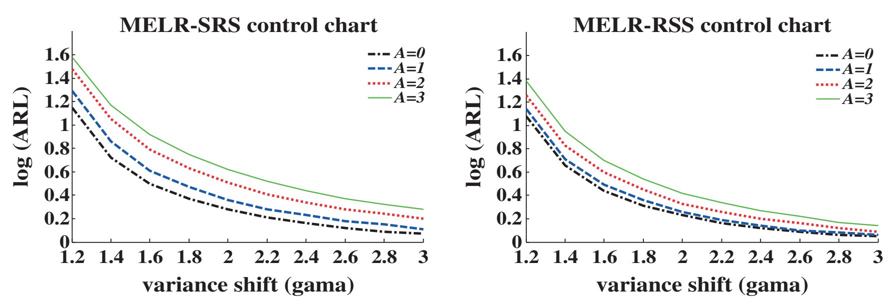

Figure 8. Values of ARL obtained by MELR-SRS and MELR-RSS control charts under variance shift for different values of A when $m = 5$, $\lambda = 0.20$, $\sigma_{\varepsilon} = 0.2$, $B = 2$ and IC ARL = 200

7. A real data example

In this section, a real data set from Montgomery[26] is used to explain the implementation of the proposed ELR, MELR and MMELR control charts based on both SRS and RSS schemes. Suppose we wish to monitor the inside diameter of the piston rings for an automotive engine manufactured by a forging process. The inside diameters are measured in millimeters (mm). First, we apply three normality tests. The *p*-values for the Anderson–Darling, Rayan–Joiner and Kolmogorov–Smirnov tests are obtained equal to 0.892, 0.10 and 0.15, respectively. Consequently, it is clear that the data set follows normal distribution and also the mean and variance of process equal to 74 and 0.01, respectively. Then, we collect data under both SRS and RSS schemes in order to provide a comparison study between them. For this purpose, three scenarios including (i) without error scenario under SRS and RSS; (ii) with error scenario under SRS and RSS; and (iii) with error scenario using multiple measurement under SRS and RSS, are considered and for each one 25 samples of sizes 5 are taken. In the first scenario (Figures 9a, 9b), the process is considered to be IC while in the second (Figures 9c, 9d) and third scenarios (Figures 9e, 9f) the process is out-of-control. Note that, in MELR and MMELR control charts under both sampling schemes, we assume that $\sigma_{\varepsilon} = 0.06$. In all scenarios considered in Figure 9, the upper control limit is chosen such that $ARL_0 = 200$ is achieved.

Base on Figure 9, both RSS and SRS schemes show that the process is statistically in control. In the second scenario, the MELR-SRS triggers an out-of-control signal at 22th sample while the MELR-RSS detects the fault in 20th sample. Similarly, MMELR control chart under SRS and RSS schemes detects the shift at 20th and 18th samples, respectively. Therefore, utilizing RSS scheme improves the capability of both proposed MELR and MMELR control charts in detecting the out-of-control signal.

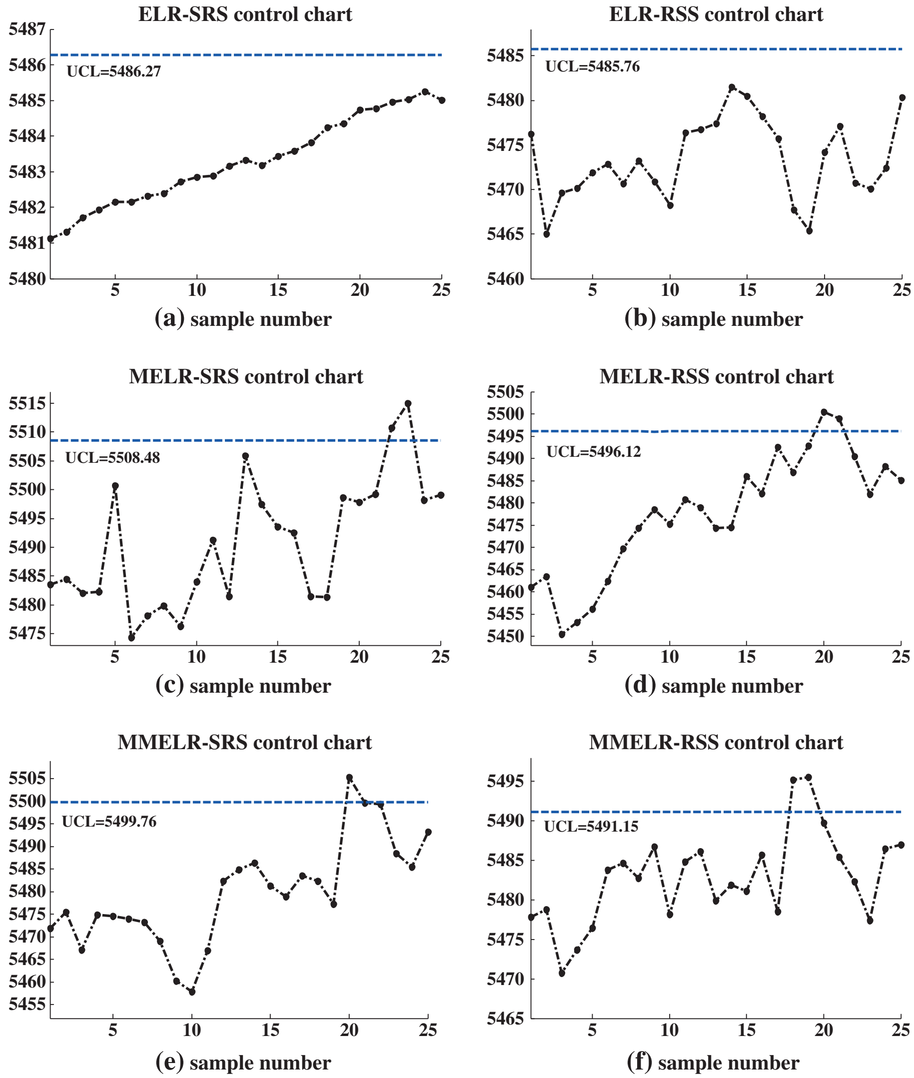

Figure 9. Comparison of the ELR, MELR and MMELR simple random sampling (SRS) and ELR, MELR and MMELR ranked set sampling (RSS) control charts under real data

8. Conclusion and future researches

Nowadays, process practitioners are intended in simultaneous monitoring of both process mean and variability. On the other hand, most control schemes provided in the literature for monitoring different manufacturing processes are presented based on the assumption that the measurements are error free. However, in most real systems, the laboratory measurements are affected by some uncertainties because of the measurement errors. In this paper, first we enhanced the ELR control chart which is presented in the literature for simultaneous monitoring of process mean and variability by utilizing RSS procedure in order to improve the performance of the control chart in detecting various out-of-control scenarios. We proved that utilizing RSS procedure instead of SRS can lead to satisfactory results in detecting shifts in the process parameters. Then, we investigated the effect of measurement error on detecting performance of the ELR control chart under both SRS as well as RSS procedures. The results showed that the measurement error can affect adversely the detecting performance of the control chart in detecting various out-of-control scenarios. We also showed that RSS procedure can cover the effect of measurement error in detecting different out-of-control scenarios and can be applied as an effective remedial approach. As the second remedial approach, we suggested multiple measurements at each sample point when both SRS and RSS procedures are used. We also provided a sensitivity analysis to investigate the effect of covariate model parameters (parameters *A* and *B*). Finally, the application of the proposed control charts is illustrated by a real dataset. For future researches, one can investigate the effects of measurement error on MAX-EWMAM control chart which is one of the most common approaches for simultaneously monitoring of process mean and variability. In addition, investigating the trade-off between statistical and economic features of the proposed ELR control chart based on RSS approach can be a fruitful area for future research.

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